MATHEMATICS EDUCATION RESEARCH GROUP OF AUSTRALASIA, INC.

MERGA Feedback: Draft Senior Years Australian Mathematics Curriculum

July 30th 2010

Introducing an Australian Mathematics Curriculum is an ideal time to address problems associated with increasing engagement in the learning of mathematics, increasing the numbers of students taking up careers in Science, Technology, Engineering, and Mathematics related fields, and improving the mathematical literacy of the Australian community. Section 3. 4.2a (p. 8) of the Curriculum Design Paper states: "In developing national curriculum for particular learning areas the *National Declaration on Educational Goals for Young Australians* is a key reference point. The second goal is that "All young Australians become successful learners, confident and creative individuals, and active and informed citizens" and the declaration states that curriculum will be "designed to develop such learners." The Senior Years Draft Curriculum Document has not ddressed these key ideas consistently throughout the curriculum document. Making appropriate statements at the start and not following through in the detail is not enough. Australia does not produced as many mathematicians and mathematics teachers as they might if the teaching and learning of mathematics were more innovative (see Tytler, et al; 2008).

The Mathematics Education Research Group of Australasia commends the intentions of the Framing Paper and Shaping Paper papers and many of the intentions of the Curriculum Design Paper. The Framing and Shape Papers stated that the curriculum should:

- Provide clear specification of what must be taught;
- Be futures-oriented;
- Clearly communicate the "big ideas" of mathematics;
- Promote depth and interconnectedness of understanding rather than superficial coverage of too many topics;
- Respect equity considerations in engaging more students, more successfully, in the study of mathematics;
- Emphasise the relevance of the content to students by drawing on their personal experience in real life contexts;
- Embed digital technologies as learning tools that deepen mathematical understanding.

The content as described in the draft Senior Years Document ***lacks***:

1. A *grand unifying idea* to underpin each course.
2. *Alignment* between rationale, aims and content, and with the Shape Paper.
3. A common statement at the start of each Rationale that captures the intent of the Shape Paper, then highlights the distinctive purpose of *that* course and the grand unifying ideas that guides selection and sequencing of content.
4. Aims for each course that are structured and sequenced to increase opportunities to employ the proficiency strands (*understanding*, *fluency*, *reasoning*, *problem solving*), with the suggested addition of *communication*.

Recommendation 1:
Make connections between the front end (Rationale, Aims, etc) of the curriculum document and the detailed descriptions so the document reflects teaching and learning of mathematics appropriate for the 21st century.

Recommendation 2:
Provide further clarification of meanings of terms teachers and mathematics educators are using differently across and within states. Define each, use them only where they apply, and embed them in the content descriptions alongside illustrations that give meaning.

Some of the key words interpreted variously are 'understanding', 'concept', and 'applications'.

a) 'Understanding' (as defined in the proficiencies) occurs as students work with mathematical ideas and struggle to make meaning. Understanding is not delivered through teacher transmission. The following quote from one MERGA member captures well the frustration at seeing statements in the preamble ("Students will be taught to understand concepts, acquire skills and solve problems related to:" (p. 3 Essential Mathematics)") that suggest understanding can be transmitted. The MERGA member commented: "The preamble to the content includes that students *will be taught to understand concepts, acquire skills and solve problems*. Can you teach these things or do you teach for them?" Such wording needs revision.

b) 'Concepts': this term is used often in situations that are not clearly linked to deep connected understandings of mathematical relationships but rather as where the term 'topic' would be more appropriate. This term should be limited to discussing the 'big ideas' / insights involved.

c) 'Applications' for some MERGA members mean applying the same procedure in a different context, and for others means adapting and or connecting procedures to solve related but different problems.

Recommendation 3:
Add 'communicating' as an additional proficiency and embed the proficiencies in the content descriptions.

Instead of dry content, students should be working with mathematics in ways that elicit interpretation and decision that help to make meaning of mathematical ideas and build deep understandings.

Although it was stated that "The proficiency strands of understanding, fluency, reasoning and problem solving have been integrated into the content descriptions, as in the K -10 curriculum" (p. 1), this has not yet been realized in any of the curriculum documents.

Illustrations of how this can be achieved are included. Essential Mathematics has been selected for the illustrations because they also shows how previously learnt content can be worked differently to reduce boredom associated with learning mathematics already covered, or anxiety associated with still not being able to understand. Included are two of many ways in which proficiencies can be embedded in curriculum descriptions: a) through mathematical modelling which can make a genuine and in some cases almost a unique contribution to meaning making, and developing understandings whilst proficiencies are employed; and b) through rewording content to include more verbs, judgments, and decisions.

a) Modelling has Proficiencies Embedded
With respect to implementation, mathematical modelling can be represented as a cyclic iterative process, and as such represents a particular type of *problem solving* [proficiencies in italics] – one in which a solution must meet non-mathematical criteria associated with its real world setting (involves *understanding* associated with interpreting; *reasoning* associated with making judgments), as well as mathematical integrity (involves *understanding*). The procedure includes specific sections directed to defining a mathematical problem (recognizing appropriate mathematics which requires *understanding*) from a real world situation, setting up a solution process (*problem*

solving), solving (*problem solving, fluency*) then interpreting the mathematical solution in terms of the problem situation (*understanding*), and finally evaluating the outcome (*reasoning*), and either returning to the problem challenge (judging / making decisions*: reasoning*), or communicating the result in clear and defensible terms (*communicating* required additional proficiency). This cyclic process is accepted internationally, and continues to be applied by practitioners in many countries, both professionally and in providing a well-structured teaching approach within educational settings. It is teachable and learnable. Modelling is applicable to many of the areas of mathematics across the four courses and more emphasis on modelling situations is required.

In particular, Courses B and C should be linked more closely to the Framing Paper (have a more of a futures orientation) through employing modelling. While there are some opportunities to develop mathematical models in these courses, such activity needs to be made transparent. This will allow teachers to show the relevance of the mathematics, and to build necessary mathematics in interesting and novel ways. Although the original Framing and Shaping Papers suggested the curriculum would encourage more students to continue their study of mathematics, this curriculum draft for all four courses is highly unlikely to do so. Modelling activities will go a long way towards changing this.

b) Illustrations of Rewording to Embed Proficiencies

E.g. 1., Essential Mathematics (Measurement: Perimeter, Area revised excerpts)

i. Recognising perimeter as appropriate for calculating such measures as length of fence, distance travelled, number of ‘rounds’ required to walk the given distance, how many ‘rounds’ were run if a given distance was travelled?

ii. Investigate the reasonableness of each perimeter formula given the shape of the ‘figure’ involved. Pathways students might take include: measuring the length around several specific examples and comparing result to answer from formula; broadly vary specific examples measured and compared with formula; or arguing why the formula has a particular structure. Students should share processes used to help interconnect ideas and increase understanding of the nature of the formulas, why they work, and how they can be considered using various methods.

iii. Automatic recall and use of formulae for perimeters of different shapes.

iv. Find perimeters of shapes for which formulae have not been given.

v. Identifying what other information is needed to calculate a perimeter when insufficient information is given and explain why this will help.

vi. Explore the areas of different shapes with the same perimeter, summarising findings.

vii. Undertake an investigation requiring the manipulation of variables to optimize a situation. E.g., design (say) a bedroom, house and yard, or recreation area, given certain financial and resource constraints to try to find an optimal product (from the student’s perspective) given the constraints.

Some examples of where Proficiencies are employed in i-vii are now listed: ‘Understanding’ is required for i), iv), and v), ‘Fluency’ for iii), ‘Problem solving’ iv), v), vi), vii), and ‘Reasoning’ for ii), iv), v), vi), and vii).

Further examples of rewording for Essential Mathematics are included in Appendix 1.

A key could be used beside different parts of the curriculum content descriptions to show the proficiencies they address. Descriptions in all four courses could be adapted to include proficiencies in ways similar to the illustrations above and in Appendix 1.

Illustrations could equally have been developed for other courses.

Recommendation 4:
In the content descriptions, identify and describe the 'big ideas' associated with each topic, and the types of mathematical activity that can developing these big ideas.

At present these lists of content are:

- Fragmented and too numerous

They should explain:

- What type of mathematical knowledge is valued in this course and why;
- What mathematical skills need to be mastered and demonstrated by hand versus by technology;
- How the content is to be treated (e.g., via practical applications only, with an emphasis on formal reasoning).

As stated in MERGA's previous feedback for the Senior Secondary Mathematics Curriculum (September 2009), the Curriculum Design Paper (June 2009) advocates in depth study, and sequencing of the curriculum:

> "The selection of curriculum content will provide for rigorous, in-depth study, preferring depth to breadth wherever a choice needs to be made"

And that:

> [The curriculum] be developed to ensure that learning is appropriately ordered and that unnecessary repetition is avoided"
> (Curriculum Design Paper. p. 18)

This has still not been addressed despite the inadequacies of the present commonly used transmissive teaching practices. It is important that the Senior Years Curriculum not remain silent on what it means to understand deeply.

In their previous feedback, MERGA stated "As there is a commonly held belief amongst many senior secondary mathematics teachers that in depth study of mathematics consists of being able to apply sophisticated rules and procedures, descriptions of the mathematical content for senior secondary years needs to be extended beyond the present content focus". This has not yet been adequately addressed.

Recommendation 5:
Reduce the amount of content in each course by about 20% to give time to the development of deep understanding by working with the mathematics through employing proficiencies rather than only using rules and procedures.

Doing so will highlight the use of mathematics to support critical thinking: building mathematical arguments; making conjectures, and investigating those conjectures. The proficiency strands need time to develop. The courses are so full of content that there is not sufficient time for the type of deep learning necessary to occur. At present, if students have opportunity to reason, and to problem solve, is not transparent.

Recommendation 6:
Include more detail in the preamble for each course about the types of content involved, its relevance for various career pathways, and the time devoted to that topic.

Recommendation 7:
Give teachers more leeway in sequencing the content to fit the needs of their student cohort by setting the content (with proficiencies) for each semester and adding a proviso that in justified circumstances, sequencing of topics could be modified.

It is important for students to see that mathematics is more than the sum of its parts. To do this, teachers need to be able to develop links between the content strands and to give the students the opportunities to build understandings. With the content arranged into semester blocks this can sometimes be very difficult to do. Teachers need the leeway to build sequences more meaningful to themselves and their students as appropriate?

Although the allocating of content by semester is probably related to giving students opportunity to change courses, a teacher who makes a connected meaningful sequence that differs to the topics as set per semester is providing other advantages that might assist a student trying to catch up mathematics they have missed.

Recommendation 8:
Give further consideration to curriculum content and sequencing with emphasis on including topics that fit well together and are likely to provide opportunities to engage students in the learning of mathematics:

Justification for content selection needs to flow from rationale and aims. It does not at present.

- All courses should have time allocated for investigation, modelling, problem solving (achieve this by removing content).
- Course A should allow flexibility for teachers to select and sequence content to meet the diverse needs of these students.
- Course B would gain coherence and reduce content if concepts were disentangled from contexts (e.g., use financial contexts for learning about growth and decay).
- Courses C & D could reduce content by deleting non-essential material and making better links between topics.

Recommendation 9:
Provide a course that is exciting and engaging for all and increases retention in mathematics courses, and builds the ability to think creatively.

- The C and D Courses are designed to be taken by the best and brightest students. It appears the the focus has been only on what **content** it was considered these students should know and the curriculum written accordingly. This is only one of the important ideas that should be taken into account when designing these courses. Another perspective that is just as important is: *What mathematical experiences do we want our best and brightest mathematics students to have?* These are the students we want to continue with mathematics and be fascinated by mathematics after they have completed these courses. As the courses currently stand, they may have the 'right content', but most students will complete these courses and be left with feeling that mathematics is dull, boring and irrelevant. They will move on to university without a passion for mathematics or in many cases, a desire to continue studying it. MERGA members would rather our best and brightest students experience mathematics as vibrant, fascinating and useful, even if this means sacrificing some of what has previously been considered 'essential content'. A greater emphasis on mathematical modelling would help.

- The following quote from a MERGA member who is a senior secondary teacher captures the importance of this recommendation: "Students (good students too) ask the question "Where am I going to use this?" when they do not see the relevance of the content they are learning especially if the course is heavily scripted and there is little leeway for the development of conjecture and the associate investigations. However, even if it is not directly relevant to them personally, if a student is able to see that the mathematics has relevance outside of the classroom and outside of the textbook they are more prepared to engage and build understandings. While the syllabi provide some opportunities, the Courses B, C and D need to provide more opportunity for this to occur."

Recommendation 10:
Shift technology from its present position as almost an optional extra (in some courses) to a powerful tool that enables the exploration of more complex situations and supports the development of deep understandings of mathematics.

As different states have progressed varying distances in their skills with using technology, and a conceptual understanding of the usefulness of technology, this area of the curriculum will need to progress over time. Right from the start though, the power of technology should be explicit in the curriculum documents.

- All courses have a "Use of technology" statement – *Technology can aid in developing skills and allay the tedium of repeated calculations* – that is inconsistent with Shape paper and with current research.
- Variable messages exist about the role of technology across courses
  - *Assumed*, *Vital* (Courses A & B);
  - *Should be widely used in this topic*; *can be used to illustrate practically every aspect of this topic* (Courses C & D);
  - No mention at all in some topics (Courses C & D)
- Implication is that technology is only valuable for less able students and only for getting "the answer".

Here it is noted that the development of ICT approaches greatly facilitates the use of real world data, which at times are messy, so that mathematical modelling and technology are symbiotic partners in education.

The following quote from a MERGA member problematises the use of technology and thus points to the types of things that need to be addressed in the curriculum documents to support teachers and educators from all states through the transition to more discriminating and intensive use of technology:

"The potential differences nation-wide in the use of technology for the purposes of assessment has implications for the time available for teaching content. For example, one of the content objectives is:

- Finding maximum and minimum values, stationary points, points of inflexion, axes intercepts, and determining convexity. In States that don't allow CAS technologies to address this point, only traditional calculus methods can be used. There are some topics where exploration and concept development will be severely limited and reduced to time-consuming procedural routines, (e.g., matrices, as a result). In [some states] …, because there are Calculator Free and Calculator Rich sections in examinations and tests teachers have to teach both traditional methods and how to use their calculator to graph, solve, find derivatives and so on because the question might be in either section. More teaching time is therefore needed for teachers in these states to address all these objectives."

State-Specific Comments
WA: Overall the proposed courses in mathematics will reduce the flexibility and options of Western Australian students seeking to study mathematics in the senior years of schooling. The State's existing courses provide more pathways and include provision for students with limited achievement in mathematics to Year 10. These students are not well catered for in the Essential Mathematics course.

The National Specialist Mathematics Course includes content which forms part of first year university work and is generally more challenging than the existing Western Australian equivalent course. As such it is more selective. Enrolments in the Specialist mathematics are likely to decline in WA as students seek to maximise their tertiary entrance scores.

Queensland in relation to Course A and B in particular: A Queensland Mathematics Educator involved in Tertiary Preparation Programs for students in all faculties (in particular first year students) who have gaps in their mathematical knowledge, points to the inadequacy of the A and B courses for preparing students for Nursing and Business. The main issue is the lack of real understanding of algebra (and to some extent its relationship to graphing). The Maths A curriculum does not appear to cover this in great enough depth and the Maths B, for many students, goes into too much depth. Nursing and Business students need an understanding of mathematics associated with substituting into formulae (including what happens when the sizes of numeric values entered change (e.g., denominator increases or decreases in size while the rest of the formula remains constant). Such understandings provide an additional check on solutions obtained and thus reduce accidental over and under doses for example. Business and Nursing students also need to understand linear functions, and relate numerical, algebraic, and graphical representations. The ability to interpret linear functions is essential.

Suggestions About Courses:

Courses

- More flexibility needed so teachers can make decisions on appropriate sequences by tailoring courses to their cohorts. Consider adding a comment that Semester 1 and 2 in Year 11 (at least) can be reorganised as long as all topics are covered during the year.

Course A

- Suggest 'Investigations' be integrated into the other topics to increase the transparency of proficiencies
- Suggest 'Algebra' be integrated into the other topics so it is developed for a purpose.
- Increase attention to number work in Course A because students in this course often have limited number knowledge.
- Select contexts that are relevant to students. For example, nautical miles units are currently included but won't be relevant to all students where there may be other conversions that are (depending on the student cohort). Make the selection of units more flexible.
- Prepare Course A with careful consideration to current VET requirements because many will not be going on to a University education (possibly 60% of students) and of those, many will enter a VET pathway. If this course does not align with VET requirements, these students will be disadvantaged.

Course B

- There is too much emphasis on financial mathematics in the Course B (Rates and ratios, Price index numbers, Financial modelling…). This mathematics could be rationalised into two topics that have less complexity and more relevance to students in this cohort. A whole unit of work on the CPI is inappropriate.
- Course B should include some simple probability associated with contexts in which students may come across it in everyday life. Possibly instead of Linear Price index. Further study of algebra could be better represented in the course.
- The developmental progression and organisation of the graphs and networks content in Course B is useful and comprehensive. The Western Australian experience is that this content can be successfully taught to the students likely to undertake this course and that they enjoy these topics.

Course A and B

- There is a gap in the difficulty level between Course A and Course B that needs to be reduced to give more options for students who are not highly mathematically able to select from to give more opportunities to enter tertiary courses of their choice.

Courses B and C

- Presumably, in trying to ensure that there is no overlap between Courses B and C, some clumsy and inconsistent organisation of content blocks has occurred. It is almost impossible to have these two courses covering many similar 'strands' but without any overlap. Another alternative to consider is having Course B course as

"Mathematics with Statistics" and Course C as "Mathematics with Algebra" (or something similar) like the structure in NZ. The courses would then have their own 'flavour' or strength and allow organising of two parallel courses in a more coherent way. The statistics focus of Course B would be appropriate for those going to university but not in a mathematics-rich field.

Course C

- Currently the aims of the Course C focus on the learning of content as opposed to the learning of broad mathematical behaviours / processes / intentions, e.g., plan and carry through tasks, choosing and using mathematical models and methods, interpret solutions, link solutions to contexts, generalise results, critically assess mathematical reasoning and conclusions, communication methods, reasoning and results. The aims do not indicate that this course seeks to develop students' abilities to investigate/think mathematically/problem solve, i.e. complete investigations and to solve problems in a range of contexts. See Aim 3 in General Mathematics, why is something like that not present for Course C?
- Calculus applications in Course C could be expanded to include more than motion only and be more in line with the rationale and the wide range of interests of potential clients (and potentially attract more students).
- Include the log-log test for power functions because it connects well with this course, particularly if modelling has more emphasis

Course D

- The rationale for Course D states this course will emphasis application of mathematics, however its content is theoretical and lacking in application.
- The units seemed to be randomly grouped. Development of topics in a more coherent and logical way that includes a link with Course C would assist teachers and students. This means realigning the content in Course C as well.
- The cognitive demand of the Year 11 Course D is significantly greater than that of Year 10. Students will move from a fairly broad based course to study of specialised topics. The current groupings appear fairly random. It should be possible to introduce and develop topics in a more coherent way following on from Year 10. The link or pathway from Course C should be included so ideas are developed logically.
- Consider omitting statistical inference, and matrices from this course to reduce the content without taking out topics that fit well together

Courses C and D

- In Courses C and D, there far too much detail on content and little identifying of 'big ideas'. It could help to divide the content description section up with a section that identifies what we want the students to understand and then a section of learning experiences that teachers could use with the students to support the development of these understandings. For example: It is not expanding $(1+x)^4$ by hand that is important but (for example) the big ideas behind being able to generate any term from any sequence $(a+b)^n$. Working in various ways with $(1+x)^4$ would then constitute a part of the learning experience. Another example: it is not the remainder theorem that is of prime importance but rather what we want students to understanding about (for example), the zeros of the function, and why they assist in finding a product. One method of exploration is through use of technology: graph the functions, identify the zeros, and then attempt to build factored equations. Students could also think about other ways to find factors, and reasons why knowing these factors can be so useful. The understandings involved through approaching this problem by a variety of methods is likely to build deep and connected understandings. Another example can be found in Calculus 2 curve sketching – explored using technology. Understanding of asymptotes and points of discontinuity are interesting and useful when considering the reasonableness of solutions to problems developed through tables, graphs, algebraically, and the physical context so links can be made between various representations, and limitations that need to be placed on models can be more clearly understood.

Assessment:

Although assessment is not part of the feedback expected at this stage, MERGA see it as essential to consider this when writing the curriculum document.

As previously stated in MERGA's September (2009) feedback to ACARA "Barnes, Clarke, and Stephens (2000) found that changes to assessment at the senior level drove pedagogical change. Thus, by connecting the assessment of the Senior Years Mathematics Curriculum to the proficiency strands (understanding, fluency, problem solving, reasoning, and communication) there is increased likelihood that the teaching and learning of mathematics at the senior secondary will shift towards student controlled mathematical thinking and collaborative development of new knowledge including *students*:

- Selecting appropriate mathematics to use
- Making decisions about the reasonableness of mathematics generated
- Making judgments about elegant ways to proceed
- Synthesizing mathematical ideas and concepts to develop insights.

These essential elements that are presently not transparent in the Senior Years Mathematics Curriculum Draft.

In Conclusion

A curriculum is "an attempt to communicate the essential principles and features of an educational proposal in such a form that it is open to critical scrutiny and capable of effective translation into practice". In describing a mathematics curriculum, the messages associated with the following stems should be articulated:

- Mathematics is …
- Mathematics is important because …
- Students learn mathematics by …
- Good mathematics teaching emphasises …
- Productive mathematics learning environments are …
- Selection and sequencing of content should be based on ….

MERGA perceive the Senior Australian Mathematics Curriculum Draft in its present form as conveying the following messages:

- Mathematics is … **cold, abstract, exclusive**
- Mathematics is important because … **eventually it can be used in other disciplines or to get a job or to gain entry to university**
- Students learn mathematics by … **practising procedures**
- Good mathematics teaching emphasises … **acquisition of rules and techniques**
- Productive mathematics learning environments are … **quiet (no discussion), individualised, almost free of technology**
- Selection and sequencing of content should be based on … **what the curriculum writers think is interesting or important.**

Instead the curriculum message should fit with the Shaping and Framing and Curriculum Design Papers and with latest research on how to engage students in the learning of mathematics to: strengthen Australian workforces of the future; produce a mathematically literate Australian community able to analyse, interpret, and creatively use mathematics to enrich all aspects of their lives; develop the 'ideas workers' of the future to keep Australia on the cutting edge in research and design in Science, Technology, Engineering, and Mathematics; and provide the much needed mathematics teachers of the future. Such Mathematics differs greatly to what is offered in the Senior Years Mathematics Curriculum

Draft. Australia needs the Senior Years Mathematics Curriculum to prepare students for their lives in the 21st Century by allowing them opportunities to study:

•Mathematics that is … **intriguing, challenging, accessible, inclusive**

•Mathematics that is important because … **it's useful, beautiful, and part of all humankind's cultural heritage**

•Mathematics students learn by … **expecting it to make sense, making and testing conjectures, communicating and defending their thinking**

•Mathematics taught by 'good mathematics teachers' who emphasise … **understanding and connecting and applying concepts and techniques**

•In productive mathematics learning environments which are … **interactive, collaborative, richly resourced**

•Mathematics with content selected and sequenced by decisions based on … **the values and unifying ideas expressed in the curriculum rationale and aims**

We note the expectation (Page 1, K-10), that the new curriculum will "…ensure that students are confident, creative users and communicators of mathematics, able to investigate, represent, and interpret situations in their personal and work lives, and as active citizens." While having specific curriculum implications, this statement points to a wider issue (arguably an ethical one), in that students can complete 10 or 12 or more years of mathematics study, and yet be unable to apply their mathematical knowledge to anything other than a formally presented assessment task. It would be a brave assertion to suggest that this has not applied to a significant majority of our students. The Senior Years Australian Mathematics Curriculum needs to address these issues.

Dr Gaye Williams
VP Development.
(Research evidence can be provided on request to support any claims made.)

Acknowledgements

Many MERGA members contributed in various ways to the development of this feedback. In particular, Merrilyn Goos (MERGA President) formulated useful insights which were used to develop a substantial part of this report.
Sincere thanks to those who provided contributions that have been integrated into this document.

Appendix 1. Examples 2-3 for Recommendation 3, Embedding Proficiencies.

E.g. 2., Measurement (p. 3) Area Excerpt Embedding Proficiencies

a). "a variety of practical applications of linear measure such as student heights, sports playing fields line marking costs and car dimensions" could be rewritten to highlight proficiencies by requiring judgments and decisions.

Include decisions about which sporting ground to use based on the cost of marking the lines so the same number of runners run the same distance but the shape of the track differs (*understanding* if not told how to make tracks but only the distance to cover in one 'round' of the figure; *problem solving* and *fluency* in working out how to make shape to get right length track). Add additional constraints by also taking into consideration the size of the parking lot, the number of cars that will fit (after optimizing the parking plan), and the resulting gate takings in each case. Now we have problem solving activity culminating in judgments (*reasoning* associated with coming to a decision) rather than boring application tasks without a given purpose.

b). Area (italics for additions as illustrations of adding purpose and judgments)

Select appropriate metric units *to find certain* areas for purposes meaningful to students (e.g., *to find the number of people a room will hold to decide how many to ask to a party, or the number of posters of varying sizes that can be fitted on bedroom wall*)), *include the need to use appropriate notation in elaborating on findings including* abbreviations. *Include some problems where* conversions between units becomes necessary because of the appropriate units for two different parts of the same problem.

Justify the reasonableness of the results obtained (above) using student-selected estimation of areas *involved*

E.g. 3., Measurement: Volume, Surface Area

a). Select a sphere and create four other solids with the same volume. Describe thought processes employed in achieving this. Explain to others.

If another sphere is selected, can you find the other shapes faster than before or not? Explain.

b). Repeat the process using surface areas not volumes.