Productive Mathematical Noticing: What It Is and Why It Matters

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Teacher mathematical noticing is a key component of mathematics teaching expertise and has been a focus of recent professional development efforts. In this paper, I propose and describe explicitly the notion of *productive* mathematical noticing, which surfaces from a case study involving a group of seven mathematics teachers who collaborated as part of a lesson study team at a primary school in Singapore. Two vignettes—one that happened during the planning stage, and the other taken from the actual lesson—are discussed to illustrate the notion of productive mathematical noticing.

"Teaching requires an unnatural orientation toward others and a simultaneous, unusual attention to the 'what' of that which they are helping others learn" (Ball & Forzani, 2009, p. 499). This "unusual attention" or mathematical noticing refers to what mathematics teachers see and how they understand instructional events or details they see in classrooms (Jacobs, Lamb, & Philipp, 2010; Mason, 2002; Sherin & van Es, 2003). Mason (2002) views mathematical noticing as a set of practices for improving teachers' sensitivity to act differently during teaching situations. These practices include reflecting systematically; recognising choices and alternatives; preparing and noticing possibilities; and validating with others (Mason, 2002, p. 95). Mathematical noticing is central to all mathematical teaching practices (Mason, 2002) and is essential for improving teaching (Schoenfeld, 2011). For example, teachers need to listen to students' mathematical reasoning and make sense of what they hear in order to teach in ways that build on s tudents' thinking (Goldsmith & Seago, 2011; Jacobs, Lamb, Philipp, & Schappelle, 2011; Schifter, 2001). Hence, it can be argued that mathematical noticing is an important component of teaching expertise. Most research focuses on the specificity of what mathematics teachers notice (Goldsmith & Seago, 2011; Kazemi et al., 2011; van Es, 2011). For instance, Jacobs et al. (2010) examined teachers' attending to children's counting strategies and how they decide to respond based on the specific details of children's strategies. While the specificity of mathematical noticing is necessary for changed practices, it is not sufficient because what is noticed may not be relevant to the learning goals; or that teachers may lack the necessary knowledge to respond. Therefore, it is important to characterise what makes mathematical noticing productive. However, the distinction between more productive and less productive noticing is not well developed. In this paper, a notion of productive mathematical noticing relevant to the learning goals is proposed and described using a case study.

Mathematical Noticing

There are different notions of mathematical noticing. While Mason (2002) views noticing as a set of practices that work together to enhance teachers' awareness to new responses in classroom situations, other researchers focus solely on what teachers attend to (Sherin, Russ, & Colestock, 2011; Star, Lynch, & Perova, 2011). However, the majority of researchers view noticing as consisting of two main processes: "attending to particular events and making sense of events in an instructional setting" (Sherin, Jacobs, & Philipp, 2011, p. 5). Some researchers, such as Jacobs et al. (2010), extend the notion of noticing to include how teachers respond to instructional events in order to link the intended responses

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to the two main processes of noticing. In this paper, a triad view of noticing—attending to noteworthy aspects, making sense of this information and deciding on the response—is used to examine what teachers see, and how they reason to make decisions with the aim of enhancing their instruction.

The ability to attend to noteworthy aspects in the midst of complex classroom environment lays the groundwork for improving practice. As Miller (2011) argues, the ability to see salient features relevant to students' learning in the buzz of classroom activities is a distinguishing mark of expert teachers. Attending to mathematical aspects of students' strategies or reasoning, for example, provides teachers insights into students' thinking and is critical in building a mathematically-rich classroom (Jacobs et al., 2010). One promising way of developing this vision involves the use of video clips of teaching—where teachers are shown clips of classroom teaching and asked to notice certain features of the instruction (Jacobs et al., 2011; Kazemi et al., 2011; Miller, 2011; Sherin, Russ, et al., 2011; Star et al., 2011; van Es, 2011). However, marking instructional events that are critical and discerning observations that are useful can be very challenging. In a video-club study involving 30 pre-service teachers, Star et al. (2011) found that the teachers had problems noticing details about the lesson tasks and mathematical content. Furthermore, teachers need to distinguish between relevant and irrelevant details in order to improve their teaching. But how this can be done remains unclear. Therefore, this paper addresses a way to make this distinction.

Besides knowing what to observe and attend to, mathematics teachers also need to develop skills in interpreting these noteworthy features. van Es (2011) highlights the importance of teachers reasoning about critical teaching and learning issues based on evidences gathered from observations. She contrasts extended noticing to baseline noticing in terms of the ability to provide interpretive comments that refer to specific events and interactions in the classrooms as evidence. Likewise, Jacobs et al. (2011) examined teachers' reasoning of children's mathematical understanding based on the alignment between teachers' interpretation of the details of specific strategies with the research on children's thinking. More importantly, these two studies position the processes of attending to and making sense of instructional events as impetus for making appropriate instructional decisions. This ties in with Mason's (2002) idea that noticing should bring to the mind of teachers, a different way to respond, and thus provide a means to examine the productivity of noticing. As Jacobs et al. (2011) contend, these three skills—attending, making sense and deciding to respond—are interdependent and work together. Thus, a triad view of noticing is useful when considering the notion of productive mathematical noticing.

In many of these studies, the specificity of what teachers notice is used as a measure of their noticing expertise (Jacobs et al., 2011; Sherin, Russ, et al., 2011; van Es, 2011). However, the specificity of what teachers notice while necessary, is not sufficient for improved practices. This is because teachers can be very specific about what they notice without having a different act in mind. This study, therefore, attempts to go beyond the specificity and explores what makes noticing more productive. More specifically, the notion of productive mathematical noticing is characterised in this paper.

Productive Mathematical Noticing

I propose a characterisation of productive mathematical noticing that builds on Santagata's (2011) notion of noticing and Sternberg and Davidson's (1983) processes of insight. More specifically, I posit that productive mathematical noticing by teachers generate insight in three ways: (1) attending to relevant information from irrelevant one that could potentially lead to new responses; (2) relating this relevant information to prior experiences to gain new understanding for instruction; (3) combining this new understanding to generate *different* possible responses to instructional events.

Santagata (2011) includes the idea of "generation of new knowledge" together with processes of "selective attention" and "knowledge-based reasoning" (p. 156) to emphasise how mathematical noticing can lead to generation of alternative teaching strategies. This is similar to Mason's idea of noticing as a means to break away from set patterns and consider appropriate alternatives during instruction (Mason, 2002, 2010). On the other hand, Sternberg and Davidson (1983, pp. 53 - 54) proposed that exceptional insight ability is a hallmark of giftedness and theorised three processes of insight—"selective encoding, selective comparison and selective combination". They illustrated these processes by reflecting on how gifted professionals such as doctors and lawyers work—sift through huge amount of information to differentiate the relevant from the irrelevant ones; compare and relate this relevant information with prior knowledge or experience; and combine this information in a meaningful way to make a diagnosis or a case. These processes mirror how teachers work in the classrooms. Hence, it is reasonable to conceptualise productive mathematical noticing as a component of teaching expertise in a similar way. To sum up, teachers notice productively when they sift out, relate and combine relevant information to generate alternatives for responding to instructional events.

The proposed notion of productive mathematical noticing hinges on the relevance of information and this is highly dependent on the context of the instructional event—what is productive in one situation may not be in another. A useful way to characterise the relevance of information is to relate mathematical noticing to the learning objectives of lessons (Santagata, 2011; Yang & Ricks, 2013). In this paper, I use the 'Three Points Framework' by Yang and Ricks (2013)—"Key Point", "Difficult Point" and "Critical Point" (p. 55)—as a lens to examine the productivity of mathematical noticing. According to Yang and Ricks (2013), the *key point* refers to the key mathematical concept or idea of the lesson and the *difficult point* is the obstacle that students might encounter in their learning of the key point. The *critical point* then refers to how students can overcome the difficulty to achieve the objective. In this paper, I address how these three points can be used to characterise the notion of productive mathematical noticing of teachers.

Method

This paper reports on two vignettes drawn from a case study—which formed part of an exploratory study—involving a six-week lesson study cycle (Fernandez & Yoshida, 2004) situated in a Singapore primary school. Seven mathematics teachers and a school leader formed the lesson study group that explored the teaching of fractions for Primary Two students (aged 7 to 8). Four of the teachers have more than 10 years of experience and the others have at least three years. The school leader, Cathy, served as an advisor to the teachers during the first four lesson study sessions. Two of the more experienced teachers—Ann (25 years) and Betty (16 years)—are of particular interest in this paper.

The study adopts an experimental model to teaching (Hiebert, Morris, & Glass, 2003) as a systematic approach to learn from teaching. In this theoretical model, teachers view lessons as experiments to examine and make sense of their teaching in order to improve their knowledge and practice (Hiebert et al., 2003). The use of lesson study in this project, therefore, supports the underlying assumption that the key to learning to teach is the ability to plan lessons that are aligned to specific learning goals and monitor the effectiveness of the lesson based on evidence collected during implementation. The lesson study protocols provides ways for teachers to discuss the learning goals (key points), learning difficulties faced by students (difficult points) and how students can overcome them (critical point). In doing so, teachers' noticing can be made visible through their discussion and artifacts.

I primarily took on a non-participant observer role during the seven lesson study sessions. Data were collected and generated through voice recordings of the lesson study sessions and video recording of the lesson. The findings were developed through identifying categories, codes and themes related to what teachers noticed and noteworthy episodes were further analysed to surface characteristics of more productive and less productive noticing based on t he 'Three Points Framework' (Yang & Ricks, 2013). Similarly to how learning objectives are used by Santagata (2011), the 'Three Points Framework' not only provide a means to examine the 'selective encoding, comparison and combination' (Sternberg & Davidson, 1983) of instructional details by teachers, but also a way to evaluate the appropriateness of instructional alternatives generated.

Results and Discussion

Two note-worthy episodes—one that happened during planning and one during the lesson—are discussed to illustrate productive mathematical noticing using the proposed characterisation. The first episode, focusing on the discussion of a group of teachers led by Betty, illustrates the notion of more productive noticing during the planning stage. The second episode focuses on the mathematical noticing of Ann—one of the teachers in the first episode—during the actual lesson and is characterised as less productive noticing. While the teachers did not explicitly used the 'Three Points Framework' in their discussion, they had discussed the learning objectives, difficulties faced by the students and the approaches they thought would help students overcome the difficulties. Therefore, the corresponding 'points'—key points, difficult points and critical points—can be attributed to the issues raised by the teachers.

More Productive Mathematical Noticing: Is it one-quarter?

During the planning of the lesson, teachers discussed the use of examples and non-examples to recap the fractional notation. One of the teachers, Ann, showed two rectangles—one was divided into four equal parts and the other was not (See *Figure 1*).

Figure 1. Representations of example and non-example of 1/4.

The key point of this segment, as identified by the teachers, is understanding the fractional notation *a/b*, where *b* refers to the total number of equal parts in the unit (whole) and *a* refers to some designated number of these parts. The teachers hypothesised that students might miss the concept of equal partitioning of the whole and how that relates to the denominator. They also discussed that it was critical to help students understand the notation by connecting it to some geometric representation.

To contrast the idea of equal partitioning, Ann used a detachable piece of the shaded part to show students the meaning of 1/4. She removed the first shaded part and compared to the rest of the parts to show that they were equal and hence the shaded part was shown to be 1/4. She then took another detachable piece (of the same area) in the second whole and went on to show that it was not 1/4 of the second whole *because the second whole was not divided equally*. Betty then raised a point of clarification:

Betty: If you take the same piece, the same piece is still ¼ of that whole.
Cathy: This is still ¼ of the whole… this one is not, but no…it's still ¼ of the whole.
Betty: Yes. You must take the small one or the big one. It's still ¼. Because it's equivalent fraction, you can subdivide that…
Ann: I don't know… make up your mind. Take or don't take?
Betty: It is still [1/4 of the whole]… you must take something that is not equal to ¼. Because that is still ¼ of the whole.
June: … yes… yes… yes… It's still ¼!
Ann: So, take or don't take?
Betty: You still take. But you must take a smaller or bigger piece. It's the same whole. It's still ¼, only that we have shifted it in a way…
Ann: Where? It's not equal, right?
Cathy: [Cathy showed the piece physically and compared it to the other whole which is not divided up equally] because this piece is still ¼ of this whole…
Ann: Oh…I see.

The error involved is a very subtle one—that the process of dividing a whole into 4 equal parts give rise to an object that is ¼ of the whole and that object can have many different pictorial representations, which remains to be 1/4 regardless of any division of the same whole. First, Betty's noticing can be characterised as productive because she was able to (1) identify Ann's subtle error which is relevant to the key point; (2) compare what Ann said to her knowledge of using areas to represent fractions; (3) combine this piece of information and relate back to the key point of the segment by suggesting that Ann should take "the smaller or bigger piece". This error was not trivial and other teachers such as Cathy and June also struggled with the concept initially as seen from their initial hesitation ("This is still ¼ of the whole… this one is not, but…"). Hence, Betty used her insight and sifted through the different pieces of information in the discussion to isolate the error. More importantly, her observation led the whole team to reinforce their understanding of the fraction concept and notation.

On the other hand, Ann's noticing was less productive at the beginning as she seemed more concerned about "taking or not taking the piece" and seemed to assert that the piece was not equal to ¼ of the second whole—"Where? It's not equal, right?". This comment is indicative of her inability to identify the issue that is relevant to the key point at first and her responses could also be attributed to her over-emphasis on e qual partitioning. However, Ann's noticing turned productive when she noted Cathy's gesture and compared the piece's area in both wholes to realise the equality of the part. Hence, she was able, at the end, to make a change in the piece for the second whole. This episode shows clearly how teachers' noticing can be directed to become more productive through discourse in a collaborative setting. This episode and several others that occurred during this study also highlight that sense making is not automatic and can be difficult even if teachers attend to the right things.

Less productive noticing: One sixth, one seventh and one eighth

The key point of this important segment is also the key point of the whole lesson—to be able to compare unit fractions by looking at the denominator. The teachers identified that students might have problems in figuring out the relationship between the denominator and size of unit fractions (Difficult point). This, in part, could be due to students bringing in their ideas of whole numbers (e.g. 8 > 7 > 6) inappropriately when they are making sense of fractions (e.g. Is 1/6 bigger or smaller than 1/8?). Therefore, the teachers planned to get students to reason or argue mathematically why a larger denominator corresponds to a smaller unit fraction in order to help them make the connection needed (Critical Point).

This episode took place after Ann, one of the teachers in the first episode, guided her students to compare two fractions 1/6 and 1/8 using the fraction discs. She wanted students to argue why 1/6 is bigger without using the physical discs. Ann guided students' reasoning through a series of directed questions during the actual lesson even though the students were supposed to articulate their reasoning according to the lesson plan.

Ann: Let's look at the whole part. How many 1/6 do I need to make a whole?
Students: Six parts.
Ann: How many 1/8 do I need to make a whole?
Students: Eight parts.
Ann: So, 1/8 needs 8 parts and 1/6 need?
Students: Six parts.
Ann: So, the more parts you need, what happen to the size of the fraction?
Students: [Silence]
Ann: The more parts you need?
Student: [Some hesitation] 1/8?
Ann: The more parts you need? What happen to the size of the fraction?
Student: [Pause] smaller?
Ann: Smaller! Yeah! When there are more parts, the fraction becomes smaller. If there are fewer parts, the fraction is?
Students: Greater.

It seemed to Ann that she was successful in getting her students to see why 1/6 is greater than 1/8. However, she soon encountered problems when she asked them to compare 1/7 and 1/8 and the following exchange took place:

Ann: One more time. How many eighths do I need to make a whole? How many sevenths do I need to make a whole? Which is more? 8 parts or 7 parts?
Students: 7!
Ann: 8 parts… 8 pieces or 7 pieces
Students: 8 pieces
Ann: Ah… 8 pieces, isn't it? If you have more pieces, what happen to the fraction?
Students: … Become 1 whole
Ann: What happened to the size of the fraction?
Students: Same.
Ann: This is 7 pieces… maybe I use another example.

Ann's noticing in these two exchanges is largely characterised as less productive. Even though her students, in the first exchange, were able to give the correct responses to the questions, she did not see that their right responses might not correspond to students' understanding of the key point. More importantly, she capitalised on a single correct answer ("Smaller! Yeah!") and guided students to say the correct response ("If there are fewer parts, the fraction is?"). She did not consider the possibility that students could have guessed the answers to her questions and hence, missed the critical point. This could be seen from her decision to carry on without asking students to reason or argue about their answers, which was supposed to be the critical point of the lesson. Without listening to the students' reasoning, it was difficult for Ann to ascertain whether they had indeed overcome the difficult point and understood the key point.

However, Ann noticed that her students had difficulties grasping the key point when they tried to compare 1/7 and 1/8. Based on students' answers, she reasoned that they might have confusion between "pieces" and "parts", which led Ann to change the phrasing from "8 parts" to "8 pieces" to help students see what she was referring to. Even though students gave the correct answer of 8 pieces, they were unable to use it to reason why 1/7 is greater than 1/8. It seemed that the students were trying to guess the correct responses to

Ann's questions. Prompted by an earlier discussion during a lesson study session, Ann then switched to a more contrasting example after this exchange—1/8 and 1/4—to support students' reasoning and it worked. While her decision to use a more contrasting example could be classified as productive noticing, she did not go back to the "1/8 and 1/7" example after that and proceeded with the group work instead. Thus, Ann missed an opportunity to determine whether her students could indeed reason about the sizes of unit fractions, which was the critical point and hence, overcome the difficult point to understand the key point. She only realised later that students were still unable to compare two unit fractions without the physical fraction discs during the group work and discussion.

Conclusion and Implications

Taken together, these two episodes illustrate how teachers' mathematical noticing could be characterised as more productive or less productive using the 'Three Points Framework' to distinguish between their noticing of relevant from irrelevant information. More specifically, analysis of the vignettes distinguished more productive noticing from less productive ones in three intertwined processes: (1) attending to information related to the key point that could potentially lead to new responses; (2) relating these relevant information to prior experiences to gain new understanding about the difficult and critical points for instruction; (3) combining these new understandings to generate *different* possible responses that target the three points. It is clear that teachers' productive mathematical noticing has enabled them to do something practical about their teaching and this practicality can be assessed using the 'Three Point Framework'. In contrast, when teachers' noticing is less productive, they miss opportunities to help students achieve the key points. When the 'Three Point Framework' is used with the proposed notion of productive mathematical noticing, teachers can begin to examine their own noticing and work towards improving their responses to instructional events more constructively. This case study has also highlighted the potential of collaborative teacher learning in enhancing the productivity of mathematical noticing.

More studies are needed, however, to examine other possible characteristics of productive noticing and investigate how teachers can make their mathematical noticing more productive in the context of professional learning and teaching. Nevertheless, just as Sternberg and Davidson (1983) framework distinguishes giftedness, the idea of productive mathematical noticing has the potential to help mathematics educators understand what makes a good mathematics teacher effective.

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