ENGAGING THE AUSTRALIAN CURRICULUM MATHEMATICS

Perspectives from the field

Editors
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Engaging the Australian National Curriculum: Mathematics – Perspectives from the Field

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Preface

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The construction of a national curriculum is always a contentious matter in any society. Arguably, in Australia it is even more so. In a federal system, with distinct school educational authorities, the political nature of the endeavours to develop the national curriculum is subject to a large degree to negotiations and compromises between the different State and Territory jurisdictions and their federal counterparts. Often these inter-jurisdiction negotiations are marked by party political affiliations that can hinder agreements – as demonstrated by the fate of the National Statements produced in the early 1990s.

Attempts to standardise the curriculum in Australia are not new. In recent times, the focus of two governments (the Howard Government and the subsequent Rudd/Gillard governments) has been to gradually shift educational decision from the States and Territories into the Federal arena. This was evident in not only shifts in funding, but achieved through the rollout of national testing (NAPLAN), the creation of the Australian Institute for Teaching and School Leadership, which will implement national professional standards for teachers, and a national process for accreditation of pre-service teacher education programs, the development of a common national body for curriculum and assessment (ACARA), and the subsequent development of the Australian Curriculum for all schools across the country. Questions can be raised as to whether, in a very expansive nation such as Australia, educational decisions can better be carried closer to the ground. We note that in this most recent attempt to nationalise the school curriculum the debate about this issue has not been very prominent in the public arena. Rather, the debate

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seems to have focused on what form should the curriculum take and what content should be included.

Likewise, developing a curriculum is an opportunity for different voices from the whole political spectrum (e.g. progressives and conservatives), as well as those representing different special interest (e.g. teachers, academics and members of the community) to present their claims and counter claims and demands. In the case of the mathematics curriculum, this social debate includes possible differing views by mathematicians and engineers, mathematics educators, as well as professional organisations representing different interests.

Hence the debate on the national curriculum is not only found in academic circles, but also in the public domain. It is carried out in academic publications and conferences, as much as it is carried out in houses of parliament, public media including blogs and wikis, even dinner tables. Here we argue that the danger is not of having a wide ranging debate leading to contrasting views. The danger is in short-circuiting the debate and cutting it down prematurely. In particular, we assert that the debate does not cease by the publication and adoption of a curriculum, rather it should continue in a cyclical manner with future modifications of the curriculum based on experiences in its implementation. This forms the main rationale behind this collection of chapters.

The idea for this book originated from a symposium at the 2010 Australian Association of Educational Research conference in Melbourne where each of the Editors made a presentation on the topic of their chapter here. Comments at that symposium were centred on the draft mathematics curriculum and the Shape Statement, as the curriculum was not released until March 2010. Prior to the conference, MERGA's developed a well-thought-out and constructed Response to the National Curriculum consultation draft in 2010.

This Book builds on such contributions by a) engaging with the full version of the curriculum published early on 2011; b) providing a wide range of content areas and foci of the curriculum; c) considering the implications of the draft for teaching and teacher development, and d) making such voices public through the publication of this collection on the web.

The various chapters by different authors represented here do not constitute a uniform theoretical approach nor can be taken as an official MERGA stance on the curriculum. Rather, the Book represents a diversity of viewpoints and stances on the Australian Curriculum: Mathematics. Our intention was to produce a set of chapters that are research based and that would consider the implications for practice, whether this involves teaching of mathematics in school or mathematics teacher education and development. We attempted to solicit contributions from academic mathematics educators with special expertise in the relevant topics, people involved from the practice of curriculum development, and teachers from different States and Territories.

We submit this collection for further deliberation by all interested parties towards the future implementation and development of the Australian Curriculum: Mathematics towards more productive school experiences for today's students as tomorrow's citizens.

Each chapter was refereed by at least two MERGA members with special expertise in the topic of the chapter. We give special thanks to the work of our colleagues and critical friends whose contribution to the refereeing process undoubtedly has raised the quality of the arguments presented here. These include

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The Editors
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Chapter 1
The Australian Curriculum: Mathematics – World Class or Déjà Vu

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This chapter raises two questions about the Australian Curriculum: Mathematics: What practices it may inspire and what might be its contributions to the national goals of education. The first questions is about the *internal cohesiveness*, or the synergy between the curriculum's stated aims and rationale, on the one hand, and the content and its articulation on the other, and what vision of mathematics education they may inspire across the different jurisdictions which would adopt it. The second question is about its *external cohesiveness,* or the synergy between the curriculum itself and the national expectations of education and what contribution it may make to the public good in the Australian society. The chapter questions whether only lip service in the elaborations is given to the General Capabilities, Cross-curricular Priorities, and the high order Proficiencies. Similarly, the lack of focus on conceptualisation and articulation of the purposes of learning mathematics opens up the possibilities of designing school-based curricula that are far from achieving the national goals of active and informed citizens.

This chapter engages with the "Introduction" to the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2011). In this context, we use the term "Introduction" to refer to the two sections of the document called "Rationale and Aims" and "Organisation" that precede the main section called "Curriculum F-10". In this analysis we raise two fundamental questions: 1) what potential practices it may give rise to in schools and 2) does it potentially contribute to the achievement of the national educational goals in Australia? While we deal with these two questions separately, they are not necessarily disjoint. The first question is about the *internal cohesiveness*, or the synergy between its stated aims and rationale, on the one hand, and the content and its articulation on the other, and what vision of mathematics education they may inspire across the different jurisdictions which would adopt it. The second question

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is about its *external cohesiveness,* or the synergy between the curriculum itself and the national expectations of education and what contribution it may make to the public good in the Australian society.

We acknowledge that the formal curriculum is not the sole determinant of the practices of teaching and learning nor is it the sole means of achieving national goals. Talking about science education, vanden Aker (1998) differentiates between the *ideal*, *formal*, *perceived*, *operational*, *experiential*, and *attained* curriculum. In dealing with the first question, we focus on the possible relationship between the *formal* curriculum and the way it may be *perceived* by teachers. By the same token, the second question focuses on the relationship between the *ideal* curriculum and the *formal* curriculum.

In the following two sections we will expand on the rationale for each question and the tools that we will adopt for discussion.

Question 1: What practices it may inspire?

The Australian Curriculum: Mathematics asserts that it does "not prescribe approaches to teaching" (ACARA, 2011, p. 7) (nor it should, we may add). However, a formal curriculum, perhaps in contrast to a mere syllabus, is developed with the expectation that it will influence the practices of teaching, learning and assessment in a particular discipline and in a particular jurisdiction. In other words, while a formal curriculum should not dictate a single approach to teaching, a possible lack of cohesion between the aims and rationale and the content and its articulation will inevitably lead to difficulties for schools and teachers in planning and implementing the actual student experiences in the subject. In particular, pedagogical and assessment decisions in schools are informed by what they consider valuable in the subject as much as by what content is suggested (mandated by a regime of testing?) in the curriculum.

Statements such as those covered in the Introduction determine the general directions and parameters in which the practice of mathematics education might take place as a result of the implementation of the curriculum. In discussing this Introduction we adopt the view that a formal curriculum is not *only* to be taken in a *technical* sense as a list of content "to be taught and learnt" (a function usually referred to as syllabus), and not only in a *practical* sense, "how content is explored or developed" (p. 2). The formal curriculum should also be taken in a *pragmatic* sense to present a vision and rationale that informs the selection, sequencing of the content, and assist its users in the development of a pedagogy and student assessment practices that are consistent with intended purposes of the curriculum (Bernstein, 1990). Thus cohesiveness in the curriculum is important at a *symbolic level*. If the Introduction section of a curriculum is to be changed from one set of articulations to an alternative and if this change has no bearing on the content descriptions, this raises serious question on the internal consistency of the curriculum. More importantly, it may be an indication that mere lip service is being paid to the stated aims and rationale or that they are not seen as valuable enough to inform content selection and elaboration.

Prior to discussing in detail the cohesiveness of the Australian Curriculum: Mathematics, it is helpful to briefly outline the structure of the document as a whole and the role of the Introduction. The Introduction of the Australian Curriculum: Mathematics serves many functions. It presents a rationale for and aims of the curriculum, and identifies the main foci of the curriculum including Content Strands, Proficiencies, General Capabilities and Cross-Curriculum Priorities. The two sections relating to the Achievement Standards and Implications for Teaching, Assessment and Reporting are also relevant to our discussion here. Although a section entitled Diversity of Learners is also included in the Introduction, we do not discuss this in this chapter, as it will be discussed in depth in the chapter by Jorgensen in this collection. While not the main focus of this chapter, the Introduction is followed by a 34 page list of the content appropriate at each year level of schooling with some elaborations on each and Achievement Standards for students at each year level. Finally, the Curriculum contains 40 pages of a Glossary defining standard mathematical terms.

The Content in the Curriculum is understood as describing "what is to be taught and learnt" (ACARA, 2011, p. 2). It is articulated into three strands called *Number and Algebra*, *Measurement and Geometry*, and *Statistics and Probability*. The description of the content lists the concepts, skills and procedures and applications relevant to each content strand. The Introduction also describes four Proficiency strands adapted from the United States' report to the National Research Council, Adding it up: Helping Children Learn Mathematics (Kilpatrick, Swafford, & Findell, 2001). Of the five strands identified in the USA model, four were used in the Australian Curriculum: Mathematics and renamed as *Understanding*, *Fluency*, *Problem Solving* and *Reasoning*. The Introduction further identifies seven General Capabilities first identified in the Melbourne Declaration as characteristic of a world class curriculum and "that underpin flexible and analytical thinking, a capacity to work with others and an ability to move across subject disciplines to develop new expertise" (Ministerial Council on Education, Employment, Training and Youth Affairs [MCEETYA], 2008, p. 13). The Australian Curriculum: Mathematics identifies these as:

- literacy
- numeracy
- competence in information and communication technology (ICT)

- critical and creative thinking
- ethical behaviour
- personal and social competence
- intercultural understanding. (ACARA, p. 8)

The Introduction then identifies three Cross-Curriculum Priorities adopted in the development of the whole Australian Curriculum; a local priority: Aboriginal and Torres Strait Islander histories and cultures; a regional priority: Asia and Australia's engagement with Asia; and a global priority: Sustainability. It is intended that "[t]he cross curriculum priorities are embedded in the curriculum and will have a strong but varying presence depending on their relevance to each of the learning areas." (ACARA, 2001, p. 10). Some of the implications of these priorities to the mathematics curriculum are discussed in the content elaborations.

Question 2: What contribution to the national goals it may have?

If the internal cohesiveness of the curriculum is important for symbolic and practical reasons, the *external cohesiveness* is equally crucial for *policy/political* reasons. Both the "what is taught and learnt" and "how is it explored and developed" can be subjected to querying as to whether they provide strong justification with respect to the social goals and education for the public good. This engagement with national goals is necessarily an engagement with the purposes of education itself. Furthermore we contend that the formal curriculum has an equally important and wider role within the entire fabric of Australian society. As Apple (1979) reminds us, the curriculum is a political activity through and through. It legitimates what knowledge and skills are valued in society and whose voices are represented. Undoubtedly, the curriculum a society produces is a representation of its traditions and history, a reflection of its cultural identification, but equally it should provide a vision of the future and a vehicle for transformation (Kennedy, 2009). In other words, curriculum development has the two faces of Janus, one face looking to the past and one looking to the future. Arguably, the development of the Australian Curriculum: Mathematics reflects an awareness of the role of education for the achievement of national goals and that these goals are in constant state of flux. For example, the first point in the rationale for the *Shape of the Curriculum Version 2.0* (ACARA, 2010) noted, "Education plays a critical role in shaping the lives of the nation's future citizens. To play this role effectively, the intellectual, personal, social and educational needs of young Australians must be addressed at a time when ideas about the goals of education are changing and will continue to evolve." Further, the Australian Curriculum: Mathematics has been developed within a bigger process that is informed by setting national goals for Australian school education in what is widely known as the Melbourne Declaration (MCEETYA, 2008) which will be discussed below.

Arguably, questions as to purposes of education are not often taken as crucial in academic and policy discourse in education (Beista, 2010). Educational debates and practices seem to be more concerned with issues related to achievement of learning outcomes and their measurement rather than with a debate on the value of the learning. In other words, as Beista comments, philosophical questions about the curriculum are often constructed as technical and managerial ones. (Similar observation can be made about research in mathematics education, which often seems to be more concerned with how we can introduce a concept to maximise learning rather than with why and to what purpose such learning is useful). Beista provides a critique of the language of *learning* as the dominant discourse in education and suggests that this is perhaps a main reason for the disappearance of the discourse on *purposes* of education. Undoubtedly, student education is the core business of schools. However a discourse of *learning*, in contrast to one of *education*, does not allow for the examination of key questions relating to the purpose and value of education. Beista calls this shift from the discourse of education to that of learning the *learnification* of education. In such a discourse students are constructed as clients and teachers and schools as providers, while education itself is constructed as being for the individual benefit rather than for the public good.

An engagement with the purposes of any school subject is necessarily an involvement with questions of values. On the one hand many participants in the curriculum debate see the power of mathematics as lying in its "objectivity" and "universality" and in being "value free" (Bishop, 1998). On the other hand there are those who argue that mathematics is culturally constructed, and both reflects and shapes the values of society (Ernest, 1991). These disparate positions on the purposes of mathematics education are shaped by personal interests and perspectives, social and political philosophies (conservative, progressive or liberal) and the interests of disparate special groups (business, mathematicians or teachers). They may also raise questions as to the balance between personal vs. public interests. However, rather than attempting to reach some sort of fragile consensus, we suggest that a continual and robust debate about the purposes of mathematics education is essential if education is to contribute to the public good and contribute to national future plans and visions.

Beista (2010) argues that the purposes of education are not uni-dimensional. Rather he identifies three different types of purposes. First, education serves the purpose of *qualification* by providing students with knowledge, concepts, skills and dispositions to be able to engage in an action or way of life. These purposes relate primarily to work, daily living and the knowledge required for future learning.

Biesta claims that education serves a second purpose which he calls *socialisation* – assisting students to participate and become part of a social, cultural or political order. This purpose enables students to learn skills that enable them to participate as active citizens in a certain community. However it also includes the development of values and norms accepted within that community(ies). Biesta asserts that at times such purposes are achieved even though they may not be explicitly articulated (as in the hidden curriculum, for example). Lastly, Biesta identifies another set of purposes to which education contributes, once again whether by design or not, which he calls *subjectification*. In one sense, these purposes are the opposite (though not incompatible) to the previous purposes. While the above purposes aim at social inclusion as a participant of a social group, these purposes aim at developing a level of independence and a sense of identify and uniqueness—in other words a human subject. They allow challenges to the social order when necessary and the construction of alternative possibilities. They allow for innovation and problem raising as well as the solving of social problems. However, one should note that these purposes do not lead back to the individualisation of education characteristic of neo-liberal approaches (to which Biesta is strictly opposed). In this construction of the purposes to education the individual does not stand alone in the social order but as a unique participant who is "subjected to the subjectivity of the other". Hence Biesta prefers the use of the term *subjectification* rather than *individualisation*.

Engaging the Australian Curriculum: Mathematics

Engaging Internal Cohesiveness

While the identification of Content, Proficiency, General Capabilities and Cross-Curriculum Priorities in the Australian Curriculum: Mathematics is to be commended, it results in a complex curriculum. The implementation of the Australian Curriculum in the design of school curricula, including selection and sequencing of activities, actual pedagogies and assessment practices employed, is not unproblematic. In developing school-based curricula, schools and teachers need to make decisions on these varying stipulations based on their judgment of their relevant importance to the general education of the students and their understanding of their roles. Traditionally, teaching in most schools has given priority to content topics as the main curriculum organisers. However, other programs, such as the New Basics Project in Queensland, have presented curricula designed around the equivalent of the General Capabilities.

In the organisation of the Australian Curriculum: Mathematics, the Content is given priority as the main organiser of the curriculum topics, with the other stipulations included in the elaborations of the Content strands. Decisions on curriculum organisation are, of course, also subject to the value given to each aspect in the regime of testing, whether this assessment is developed by the schools themselves or imposed on them externally. Consistent with privileging the Content as curriculum, the Achievement Standards sections describe skills and concepts articulated in the content descriptions, with little or no reference to other capabilities. Without excluding the possibility of other practices, the discussion of assessment in the Curriculum seems to target the Content strand in its wider interpretation of concepts, processes and skills. No direct mention in this discussion is made to Proficiencies, General Capabilities or Cross-Curriculum Priorities.

If these capabilities are, indeed, an integral part of students' mathematics education, how helpful is the Curriculum in promoting their implementation in the mathematics classroom? First we note that there is an apparent confusion in the articulation of Proficiencies in the Introduction and the ways in which they are exemplified in the Content elaborations.

On one hand, these proficiencies are understood as "*actions* in which students can engage when learning and using the content" (ACARA, 2011; p. 2. Italics added for emphasis). In other words they describe "how the content is explored or developed" (p. 14). Hence, in this understanding, they are not presented as outcomes or aims to be developed and assessed. This view of proficiencies as means of experiences or actions, is consistent with their absence from the statement in the Introduction about Achievement Standards that "achievement standards indicate the quality of learning" which is described as "the extent of knowledge, the depth of understanding and sophistication of skills" (p. 6).

On the other hand, the actual descriptions of the four proficiency strands give an alternative construction of their nature. For example, Understanding is elaborated as: "Students build a robust knowledge of adaptable and transferable mathematical concepts"; Fluency as: "Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately"; Problem Solving as: "Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising"; and Reasoning as: "Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising"(p. 3). These articulations imply that the proficiencies describe dimensions of student performance within mathematics rather than a type of experiences they have in its study.

It is worthwhile to note that this last understanding is consistent with the interpretation in the original U.S. model. Kilpatrick et al (2001) explain "recognizing that no term captures completely all aspects of expertise, competence, knowledge,

and facility in mathematics, we have chosen mathematical *proficiency* to capture what we think it means for anyone to learn mathematics successfully" (p. 5). Here the proficiencies are taken as components of mathematics performance that can, as the curriculum writers assert, be developed through mathematics teaching. Indeed, this interpretation of Proficiency is reflected in the second Aim adopted by the Australian Curriculum: Mathematics itself that students "develop an increasingly sophisticated understanding of mathematical concepts and fluency with processes, and are able to pose and solve problems and reason in Number and Algebra, Measurement and Geometry, and Statistics and Probability" (ACARA, 2011, p. 1).

Moreover, it is important to also be conscious of the origin of this particular list of proficiencies. The authors of the US report acknowledge:

> Our analysis of the mathematics to be learnt, our reading of the research in cognitive psychology and mathematics education, our experience as learners and teachers of mathematics, and our judgments as to mathematical knowledge, understanding, skills people need today have led us to adopt a composite, comprehensive view of successful mathematics learning. (Kilpatrick, et al., 2001, pp. 115-116)

Arguably, every single framework for talking about mathematics education arises from within a particular philosophy of education and a particular epistemology. It also reflects a certain view of what is ultimately valuable in students' engagement in their school experiences. In terms of the discussion by Biesta (2010) referred to above, this model of proficiency arises within the *learnification* paradigm. It attempts to summarise the dimension of student performance that posits mathematical knowledge as an aim by itself rather than a tool to achieve other purposes. An alternative literature base, for example one that takes a more social and critical dimension such as critical mathematics, mathematics for social justice or ethnomathematics, would produce different desired and valued proficiencies and/or different understandings of these identified proficiencies.

But how are these proficiencies reflected in the Content elaborations in the curriculum itself? To illustrate a general concern about the implementation of these proficiencies, we have examined the elaborations at one arbitrary school level, year 8, and analysed which proficiencies are in focus at that level. One has to first recognise that the above four proficiencies are not disjoint, either under an understanding of them as dimensions of performance or as experiences provided by the teachers. Hence some content elaborations may relate to one or more of the proficiencies and there may be some disagreement on the interpretation of some elaborations. However, an interesting pattern arose from our analysis. Twenty three of the 43 elaborations, (53%) relate to experiences to develop understanding (e.g. "understanding that the real number system includes irrational numbers and that certain subsets of the real number system have particular properties" (p. 38)).

Twenty four of the 43 of the elaborations (56%) relate to developing fluency (e.g. "evaluating numbers expressed as powers of positive integers" (p.3 8)). Five of the 43 elaborations (12%) relate to problem solving (e.g. using percentages to solve problems, including those involving mark-ups, discounts, profit and loss and GST (p. 38)). A mere three of the 43 elaboration (7%) refer to reasoning (e.g. "investigating an international issue where media reporting and the use of data reflects different cultural or social emphases (for example whaling, football World Cup outcomes)" (p. 40)).

Perhaps the message that these might give to a teacher as to what type of experiences they should focus upon is rather obvious. However, let us examine these elaborations further. Of the five elaborations that are obviously related to problem solving, two are from within the context of mathematics itself (e.g. congruency theorems); one from financial interactions (e.g. profit and discounts) and three are open ended related to "probability questions about objects or people" and "collecting data to answer the questions about Venn diagrams" (p. 40). Of course authentic problem solving may still be used by teachers in order to bring mathematics alive and to assure enriching meaningful experiences; however it is valid to raise the concern that the curriculum document itself may not inspire teachers to appreciate the importance of these proficiencies and to think of valuable and exciting ways in which they can be used or developed in the classroom.

Similar concerns can be raised about the minimal reference in the Content descriptions and elaborations to the other stipulations in the curriculum such as the General Capabilities and the Cross-Curriculum Priorities. On our reading of the document, their incorporation in the elaborations is either non-existent or, at best, trivial. It is relevant to note that while the Proficiencies are mentioned in the Aims the Curriculum but not in the sections on assessment, the General Capabilities and Cross-Curriculum Priorities are not mentioned in either. This gives the impression that either lip service is paid to them (despite the assertion that they are important), or that they are seen as natural by-products of the curriculum and hence they do not need to be elaborated and illustrated, or that teachers may rely on sources other than the Curriculum itself to plan for their achievement. We suggest that these latter interpretations are somewhat spurious.

Hence, we are not convinced that the published Curriculum is internally consistent in its assertions of what is important in the Introduction when this is read in conjunction with the articulation of the content and its elaboration in the main body of the document. The heavy dominance in the curriculum of Content as an organiser of knowledge and as the major focus of the elaborations, and the limited explicit articulation of the effect of the other stipulations, lead to two (perhaps

related) concerns for its potential to contribute to the practice of mathematics education in Australian schools.

The first concern was raised by Luke (2010) in his article in a special issue on the Australian Curriculum in Curriculum Perspectives. Luke points to findings of a wide-ranging longitudinal study conducted in Queensland on school teaching. The study showed that most classrooms studied revealed a strong focus on what can be called core skills and content in the different subjects observed. However, the study identified as major requirements for higher student outcomes a sustained focus on high order thinking skills and a visible connection between the school subjects and the life knowledge and social life of the students. Of course, the Australian Curriculum: Mathematics does not close the door on teachers addressing these challenges in their teaching. But nor does it provide much direct help to assist teachers to realise these challenges, or useful exemplars on how they can construct "powerful and engaging experiences" (p. 1) for their students.

The second concern, or a worst case scenario, is that the Australian Curriculum may be utilised to justify a "back to basics" movement in mathematics education. For some political leaders in the highest offices of the land that is exactly what the national curriculum intends. On February 28, 2010 The Age newspaper reported on the launch of the Australian Curriculum where the then Prime Minister, Mr Rudd said the objective was,

> without apology, to get back to the absolute basics on spelling, on sounding out letters, on counting, on adding up, on taking away. The basics that I was taught when I was at primary school a long time ago, and that's what our national curriculum is all about. (Bachelard & Stark, 2010 para. 8-9)

This expectation that the national curriculum would represent a "back to basics" focus is also represented in some editorials and many letters to the editor of many newspapers around Australia. Yet, this was not the way the curriculum developers themselves saw their endeavour or aim. In the same article, The Age quotes Barry McGaw, the ACARA Chair of the National Curriculum Board as saying,

> I don't like back to basics, because it implies you're only focusing on initial performance … we need a curriculum that builds the basics, but also extends students, hence the emphasis on literature. (Bachelard & Stark, 2010, para. 11)

Of course, one can attribute the above assertion about the "back-to-basics" to mere political rhetoric. However, discourse is never innocent. As Foucault (1976) reminds us, it creates the reality it talks about. We share the concern expressed by Luke (2010) that a major danger of a curriculum focusing on content and skills is the possibility that "it sets the conditions" (p. 8) for such an eventuality. Undoubtedly, a back-to-basic movement would be contrary to viewing of educational goals as ever developing in response to changing social contexts as expressed in both the

Melbourne Declaration and the Shape Statement mentioned above. Here we argue that, if the ultimate justification of the curriculum is to form the basis for regimes of testing across the different school systems, the elaborated content is totally justified. However, for the curriculum to lay the foundation of reform in the teaching and learning of mathematics, such lack of elaborations of the higher order Proficiencies, General Capabilities and Cross-Curriculum Priorities are regrettable, to say the least.

Engaging the External Cohesiveness

The previous section discussed what we have called the *internal cohesiveness* of the Australian Curriculum: Mathematics. The main focus there was the consistency between what the Curriculum professes to be important and the way it describes the content to be taught and experienced. The main concern was with what type of classroom practices that the Curriculum might enable, encourage and embody. The discussion in this section is about its *external cohesiveness*. The focus here is the consistency between the curriculum produced and the national goals of education as articulated by the Melbourne Declaration in particular, and the discussion of purposes of education articulated by Biesta (2010). The main concern is the potential contribution of the Curriculum to education for the public good of Australians.

The Melbourne Declaration (MCEETYA, 2008) represents an agreement by all State, Territory and Federal Ministers of Education on the educational goals of Australian young people. It identifies two major goals for schooling in Australia.

Goal 1: Australian schooling promotes equity and excellence

Goal 2: All young Australians become:

- successful learners
- confident and creative individuals
- active and informed citizens (p. 7)

The Declaration goes further to elaborate on the implications of each one of these goals through a series of dot points—some of them will be exemplified below. The Introduction to the Australian Curriculum: Mathematics identifies three somewhat similar aims:

The Australian Curriculum: Mathematics aims to ensure that students:

- are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens
- develop an increasingly sophisticated understanding of mathematical concepts and fluency with processes, and are able to pose and solve problems and reason in Number and Algebra, Measurement and Geometry, and Statistics and Probability

- recognise connections between the areas of mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study. (p. 1)

Arguably, few people would disagree with the development of confident and creative users of mathematics, of a sophisticated understanding, skill and application of the content, and of an appreciation of mathematics as an accessible and enjoyable discipline as highly worthwhile aims for mathematics education. In particular we applaud the inclusion of terms such as “confident and creative users”, “active citizens”; “accessible and enjoyable discipline” as well as the mention of the proficiencies along with the content in the Aims (notwithstanding the confusion referred to above in the articulation of proficiencies in the Content descriptions and their absence from articulation of assessment). Similarly, we note the partial overlap between the adopted Aims in the Curriculum and the Goals of the Declaration. The purpose of our reflections here is neither to present a comprehensive analysis of the similarities and differences between the two statements nor to analyse whether the stated Aims in the Curriculum are reflected in the main body of content selection and elaborations. Rather our purpose is to engage with both sets of Goals and Aims with respect to what type of purposes of education (à la Biesta) they represent, and to query the cohesiveness between the two.

First we note that two of the Aims identified above (2 and 3) refer to desired purposes of learning from *within* the discipline itself (Aim 2) and the overall school curriculum (Aim 3). In particular Aim 2, simplified, states that the student should develop proficiency *in* the content. Aim 3, simplified, states that students can “recognise” connections between mathematics and other curriculum subjects and enjoy its study. By simplifying them in this way we do not mean to devalue their importance, but rather to highlight that they refer to what it means to be a successful learner in mathematics from the perspective of the subject itself and from its position in the overall curriculum. However, the first Aim refers to a desired identity of a successful leaner of mathematics as a “confident and creative user of mathematics” who, in particular, is “able to investigate, represent and interpret situations” (p. 1). It sets the ground for going beyond the learning of/in the curriculum itself to the purposes of mathematics education itself. Hence, the very articulation of Aims of the Curriculum reflects the strong *learnification* discourse in education that Biesta notes. To highlight this point, we refer to the articulation of the Goal of schooling (sub-goal “to develop successful learners”) as articulated in the Melbourne Declaration. The elaborations of that particular Goal identify “developing the capacity to learn”, “obtain and evaluate evidence”, “solve problems that draw upon a range of learning areas”, and to “make sense of their world and think about how things become the way they are” (p. 8). The focus in these interpretations of the goal "of successful learner" is on ways of being and acting as an active citizen as a result of their experiences in schools. They are less focused on the level of disciplinary learning (including proficiency in it) than they are on the vision of an educated person as a result of their school experience.

In this context we note that while the learnification focus of the Aims of the Australian Curriculum: Mathematics does not necessarily avert the achievement of the Goals of the Melbourne Declaration, neither does it set out explicitly towards their achievement. We argue that this is not merely an arbitrary linguistic variation in conceptualising aims and goals. Rather it is a crucial difference if the curriculum is to play a leading role in reforming the practices in mathematics education in Australian Schools. Traditional teaching in mathematics, as noted by Bishop (1998) in the 1990s and as still holds in many schools, has tended to concentrate on concepts and techniques with a relatively limited level of direct applications. One could argue that in recent times this focus has been encouraged by a regime of testing that is increasingly determining the agendas for many schools. A shift of focus from what knowledge and skill is required in/by mathematics and in schools to a focus on what is required for a citizen to become a confident and effective user of mathematics in society would assure that mathematics is more aligned with the conceptualisations articulated in the Melbourne Declaration.

We now turn to a reflection on the purposes of studying mathematics (in other words what is mathematics good for in contrast to the Aims of the Curriculum itself). While the Curriculum does not specifically identify these different purposes, several purposes can be identified in its description of the different components. Perhaps they can be summarised as follows, in the order of their appearance in the document:

1. Competency in mathematics is useful for everybody in their personal life, work place and civic lives.
2. The three content areas specifically contribute to practices such as financial planning, design and interpretation of data to make informed judgements.
3. Mathematics is needed for certain professions and specialists.
4. Mathematics content is useful for other school subjects.
5. School mathematics builds foundations for further study in mathematics and beyond the school.
6. School mathematics allows students to appreciate the beauty, elegance and power of the discipline of mathematics.

Once again few would contest that mathematics is an important school subject for these, perhaps among other, purposes. In many ways they are very familiar to many teachers and curriculum developers around the world. However it is worthwhile to note that these are all individualistic constructions of the benefits of

mathematics, none of which highlights the public good stemming from a quality mathematics education in schools. Here we have in mind not only the benefits of mathematics to economic and technological developments of society, but also its benefits in the identification of social problems and its contribution to their management. If this contribution of mathematics to the general good of society is taken seriously, then not only will different purposes be identified, but also the examples given in the document of activities that teachers can use would be different.

Similarly, we note that these purposes are constructed from a perspective that sees mathematics as a fixed body of established knowledge and processes whose different uses are relatively certain and unproblematic. In another context, Thornton (2011) discusses the absolutist philosophies of knowledge that seem to be behind the construction of the Australian Curriculum: Mathematics and questions if these traditional views of mathematics are sufficient to construct a curriculum that will assist students in their future lives in times characterised by uncertainty and fuzziness. If alternative views of the nature of mathematics are adopted, then, for example, the usefulness of mathematics to other areas of study may be constructed in terms of possible conflict that may arise from the use of different knowledge bases in the solutions of real world problems, rather than simply from an unproblematic assumption that mathematics contributes to the development of knowledge in other school subjects.

The technical purposes identified above are consistent with the purposes of mathematics for *qualification* of Australian students in their personal and work lives. Further, such qualification itself may encourage more effective participation in civic and social life. However, this participation is different from the s*ocialisation* purposes discussed by Biesta. Above we acknowledged that a certain level of socialisation arises from experiences of students in any pedagogical activity—either explicitly or as a hidden curriculum. Here we take the stance that an unconscious and uncritical socialisation may contribute to the hegemony of objectivity and rigour of hard sciences such as mathematics that dominates political, social and educational discourse in the 21st century. If in mathematics education the "power" of mathematics is demonstrated without questioning the detrimental effects of such power (Simmons, 1999), if the usefulness of mathematics to other disciplines is presented uncritically without consideration of a possible conflict with the values of other knowledges, or if mathematics is not used to understand social life but merely only to function in it, then the socialisation of students through school mathematics may lead to an unquestioning privileging of mathematical knowledge and of the decisions based on it. Questions need to therefore be raised as to the nature of the socialisation it is desirable to achieve through the mathematics experience of school students. Such questions can then lead to a more holistic view of the purposes of mathematics and can, in turn, be reflected in curriculum decisions in terms of the content and pedagogy needed to implement such purposes.

Finally we raise the question of whether the above purposes reflect the role of mathematics in the *subjectification* of the student. The contribution of mathematics to the subjectification of the student cannot be realised when mathematics is seen solely as a technical subject that is useful in enabling people to function in work and society. However, when an alternative language is used to identify the "power" of mathematics, it can contribute to not only a critical understanding of how the world has come to be, but also to the development of the agency to change it. In the literature, such language is used in much of the critical mathematics education, ethnomathematics and mathematics for social justice literature. Constructs of *agency*, *empowerment* and the *transformation* of society are terms that can be used to highlight the subjectification purposes of mathematics. These purposes do not imply less mathematics knowledge, but more. As Ernest (2002) argues, empowerment of students in and through mathematics necessarily includes *mathematical empowerment* which consists of the ability to critically read and produce mathematical texts as well as to pose their own problems and solve problems. This may imply a very different selection and organisation of mathematical experiences.

Here we argue that the development of mathematics in isolation from the capacities developed in other areas of school curriculum limits the role of mathematics in achieving its transformative potential. In examining the claim of the Australian Curriculum: Mathematics to have a future orientation, Atweh and Goos (2011) summarised that

> the identification of content into the traditional mathematical fields of mathematics may be convenient in a syllabus, but it does not lend itself to dealing with real-world applications that often require cross-disciplinary approaches. With the increasing focus on overall capacities in thinking about preparing students for future, it is left to teachers to see how the content can be used to develop the cross-curriculum competencies, and the higher order proficiencies identified in the Australian Curriculum: Mathematics. (p. 223)

The implications of such purposes of mathematics to the curriculum are varied. A curriculum that aims to develop student *subjectification* places, at its centre, problem posing and authentic problem solving, mathematical reasoning, and critical thinking. However, this reasoning and thinking occurs not only through developing the mathematics itself, but also developing the ability to use mathematical knowledge in context of real world problems; it must also involve learning about mathematics itself - its assumptions, power and limitations.

Concluding Remarks

By way of conclusion, we will return to the question raised the title of this chapter. Does the Australian Curriculum: Mathematics represents a world class curriculum or is it more of the same? In developing the Australian Curriculum, the discourse of "world-class" was highlighted in political-speak as well as the many documents outlining the rationale of the curriculum and its dissemination. The Melbourne Declaration understands a world class curriculum as the one which, among other things, "will … support the development of *deep knowledge* within a discipline, which provides the *foundation for inter-disciplinary approaches* to innovation and *complex problem-solving*" (p. 13) (Italics added for emphasis).

In the discussion above we noted that the Content aspect of the Curriculum is its main organiser of the experiences that student are expected to engage in and achieve in their study of mathematics at all levels of schooling. Other worthwhile aspects such as Proficiencies, General Capabilities and Cross-curricular Priorities are presented as general statements and/or elaborations of the Content. In this chapter, we have not engaged with the question of Content selection as such. Our concern was with the rationale behind the content selection and organisation that may guide teachers and schools in their construction of their school curricula, pedagogical and assessment practices.

In this chapter we raise the question whether only lip service in the elaborations is given to the General Capabilities and Cross-curricular Priorities. Even the Proficiencies are not equally represented and give the impression that understanding and fluency are more important than problem solving and reasoning. Of course, the document does not specify this explicitly. However, teachers and schools are likely to construct their school curriculum to be along the lines of the Australian Curriculum: Mathematics. Not many educators would equate a focus on understanding of concepts and fluencies in procedures with the "deep knowledge" and "complex problem-solving" expressed in the Melbourne Declaration.

Similarly, the lack focus on conceptualisation and articulation of the most important purposes of learning mathematics in the first place is of little assistance to teachers to make decisions about what aspects of the curriculum are most worthwhile in planning their pedagogy and assessment practices. This also opens up the possibilities of designing school-based curricula that are far from achieving the national goals of active and informed citizens. Such goals can be achieved through engagement of students with authentic real world problems from the social life (Atweh & Brady, 2009) that lay the "foundation for inter-disciplinary" approaches identified by the Melbourne Declaration above.

Of course, what specific practices and outcomes might the Australian Curriculum: Mathematics inspire and achieve can only be speculated. One thing we can be certain about, based on years of research and experience, is that these practices would differ between schools with diverse visions and students from different backgrounds. However, we remain sceptical about the proposition that the Curriculum in its current form does provide a world class vision of what mathematics education can, or should, be like.

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Chapter 2
Working with the Big Ideas in Number and the Australian Curriculum: Mathematics

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This chapter will explore why big ideas have become a topic of interest in mathematics education, what these might look like in relation to the teaching and learning of number in the early to the middle years of schooling, and how these are reflected in the *Australian Curriculum: Mathematics*. In particular, it will consider those ideas and strategies without which student's progress in mathematics will be seriously impacted. Identified as trusting the count, place-value, multiplicative thinking, partitioning, and proportional reasoning (Siemon, 2006), each idea will be explored through the lens of a set of diagnostic materials that have been found to be effective in mainstream and remote Indigenous settings. Two case studies will be reported to demonstrate the efficacy of the diagnostic tools and their associated advice in helping teachers identify and respond to the specific learning needs of their students while building a deeper understanding of the mathematics needed for teaching.

The crowded curriculum and the lack of succinct, unambiguous guidelines about the key ideas and strategies needed to make progress in school mathematics have long been a concern of teachers. This is particularly the case for Number which is the area most responsible for the significant range in mathematics achievement in the middle years of schooling (Siemon, Virgona & Corneille, 2001; Siemon, Breed, Dole, Izard & Virgona, 2006). While the importance of focussing on the 'big ideas' is widely recognised (e.g., Charles, 2005; Kuntze, Lerman, Murphy, Kurz-Milcke, Siller & Winbourne, 2009; National Curriculum Board [NCB], 2009: Ontario Ministry of Education, 2006), there is little agreement about what these ideas are and how they are best represented to support the teaching and learning of mathematics in schools.

The development of the *Australian Curriculum: Mathematics* [ACM] (Australian Curriculum Assessment Reporting Authority, 2011) provided an important opportunity to negotiate and articulate the big ideas in school mathematics (see

In B. Atweh, M. Goos, R. Jorgensen & D. Siemon, (Eds.). (2012). Engaging the Australian National Curriculum: Mathematics – Perspectives from the Field. Online Publication: Mathematics Education Research Group of Australasia pp. 19-45.

Siemon, 2011b). The extent to which this has been achieved is discussed in terms of five big ideas in Number that have been found to be useful in identifying learning needs and informing teacher's responses to those needs in Victorian, Tasmanian and South Australian schools.

Why a Focus on Big Ideas?

> Students need to learn mathematics in ways that enable them to recognise when mathematics might help to interpret information or solve practical problems, apply their knowledge appropriately in contexts where they will have to use mathematical reasoning processes, choose mathematics that makes sense in the circumstances, make assumptions, resolve ambiguity and judge what is reasonable in the context. (Commonwealth of Australia, 2008, p. 11)

Evidence from the 2009 *Programme for International Student Assessment* [PISA] shows that there has been a significant decline in the proportion of Australian students reaching Level 5 or above on the mathematical literacy scale (Thomson, Bortoli, Nicholas, Hillman & Buckley, 2011). This is consistent with data from the *Middle Years Numeracy Research Project* [MYNRP] which found that a significant proportion of students in Years 5 to 9 experience considerable difficulty interpreting problem situations, applying what they know to solve unfamiliar situations, explaining their thinking and communicating mathematically (e.g., Siemon, Virgona & Corneille, 2001).

While it is difficult to argue cause and effect at this macro level, there is little doubt that the opportunity to engage in sustained problem solving and in-depth investigations is significantly influenced by the amount of content that teachers feel they have to teach and how that content is offered (NCB, 2009). This is reflected in the size and organisation of mathematics textbooks in the middle years where mathematics is typically presented as a set of disconnected topics and the primary mode of learning is example-practice-practice. The fact that there is considerable overlap in the content of such texts and the vast majority of problems tend to be relatively low-level, skill-based repetitious exercises (e.g., Vincent & Stacey, 2008) is unlikely to be conducive to learning mathematics in the way suggested by the *National Numeracy Review* (Commonwealth of Australia, 2008). A focus on the big ideas is needed to 'thin out' the over-crowded curriculum (NCB, 2009; National Mathematics Advisory Panel, 2008) and create opportunities to rethink and transform existing approaches to the teaching and learning of mathematics.

> Teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise. (National Council of Teachers of Mathematics, 2000, p. 17)

A focus on big ideas and the links between them is also needed to strengthen student understanding and help deepen teacher knowledge and confidence for teaching mathematics (Charles, 2005), the importance of which has been demonstrated by research on the characteristics of effective teachers of mathematics (e.g., Askew, 1999; Charles, 2005; Clarke & Clarke, 2002; Hattie, 2003; Ma, 1999). For instance, effective teachers recognise the connections between different aspects and representations of mathematics. They ask timely and appropriate questions, facilitate and maintain high-level conversations about important mathematics, evaluate and respond to student thinking during instruction, promote understanding, help students make connections, and target teaching to ensure key ideas and strategies are understood. "A clearly, succinctly written curriculum will assist this" (NCB, 2009, p. 12).

What is a 'Big Idea' in School Mathematics?

The content of school mathematics has always been subjected to some form of categorisation. In recent times, these categorisations have included process as well as content strands, for example, the *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991). For its purposes, PISA categorised school mathematics in terms of 'quantity, space and shape, and uncertainty'. In the *Discussion Paper on School Mathematics for the 21st Century*, the Australian Association of Mathematics Teachers (2009) added 'variables, relationships and change' to the PISA list but also included four 'mathematical actions'. The ACM is organised in terms of three content strands – 'Number and Algebra, Measurement and Geometry, and Statistics and Probability' - and four proficiencies. While some of these categories (e.g., quantity, uncertainty) might be regarded as really big ideas in school mathematics, they are too broad to inform teacher's everyday practice. A more refined set of key ideas and strategies and the links between them is needed to inform teaching and scaffold student learning.

Charles (2005) defines a 'big idea' as "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (p. 10). For example, "Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value" (p. 14) is a statement of a big idea. However, while he identifies twenty-one 'Big Ideas' in mathematics and provides 'examples of mathematical understandings' for each, no claims are made about possible learning progressions or developmental priorities beyond what is loosely and perhaps unintentionally implied by the organization of the list. For example,

> Big Idea #2: The base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value. … (p. 13)

> Big Idea #13: Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found. (p. 18)

While not defining a 'big idea', the Ontario Ministry of Education (2005) lists five 'big ideas' in number sense and numeration for K to 3 – 'counting, operational sense, quantity, relationships, and representations'. For Years 4 to 6 'counting' is replaced by 'proportional reasoning' and in Years 7 to 8, the list is reduced to 'quantity relationships, operational sense, and proportional relationships'. This approach provides some indication of the developmental progressions involved but these are not specifically delineated.

More recently, the *Awareness of the Big Ideas in Mathematics Classrooms* project (Kuntze, Lerman, Murphy, Siller et al., 2009), which is aimed at "encouraging teachers' reflections on overarching concepts in mathematics and on their potential for learning" (p. 9), has identified four characteristics of big ideas. These can be summarised as ideas that have high potential for building conceptual understanding, meta-knowledge about mathematics as a science, meaningful communication strategies, and professional reflection. Examples of big ideas from this standpoint include 'using multiple representations', 'giving arguments or proving' and 'dealing with infinity'. While these are undoubtedly important indicators of mathematical reasoning and best teaching practice, it is not clear how these translate to learning trajectories that could be used to inform teaching and support mathematics learning over time.

For the purposes of the *Assessment for Common Misunderstandings* (Siemon, 2006) and the *Developmental Maps* (Siemon, 2011a) which were developed for the Victorian Department of Education and Early Childhood Development [DEECD], a 'big idea' in mathematics:

- is an idea, strategy, or way of thinking about some key aspect of mathematics without which, students' progress in mathematics will be seriously impacted;
- encompasses and connects many other ideas and strategies;
- serves as an *idealised cognitive model* (Lakoff, 1987), that is, it provides an organising structure or a frame of reference that supports further learning and generalizations;
- cannot be clearly defined but can be observed in activity … (Siemon, 2006, 2011a).

Why Big Ideas in Number?

Teachers routinely point to Number as the most difficult aspect of the school mathematics to teach and learn. This is reflected in the time spent on number in the school mathematics curriculum and evident in the data from the MYNRP, which used rich assessment tasks and partial credit items to explore number sense, measurement and data sense, and space sense in a structured sample of 6859 Year 5 to 9 students in 1999-2001 (Siemon, Virgona & Corneille, 2001). The results of this large-scale study found that there was as much difference in numeracy achievement within schools as between schools, that in any one year level there was there was up to an 8 year range in ability, and the needs of 'at risk' learners were not being met. A key finding of the MYNRP was that the differences in performance were almost entirely due to difficulties with larger whole numbers, decimals, fractions, multiplication and division, and proportional reasoning, collectively recognised as multiplicative thinking (Vergnaud, 1983).

As a consequence, the *Scaffolding Numeracy in the Middle Years* [SNMY] project was designed to explore the development of multiplicative thinking in Years 4 to 8 using rich tasks and partial credit items. Rasch modeling (e.g., Bond & Fox, 2001) was used to analyse the responses of just under 3200 students in three school clusters (one secondary school and three or more associated primary schools), two in Victoria and one in Tasmania (Siemon, Breed et al., 2006). A *Learning and Assessment Framework for Multiplicative Thinking* [LAF] was identified on the basis of this analysis comprising eight hierarchical zones ranging from additive, count-all strategies (Zone 1) to the sophisticated use of proportional reasoning (Zone 8) with multiplicative thinking not evident on a consistent basis until Zone 4. The proportion of students by Zone by Year level is shown in Figure 1.

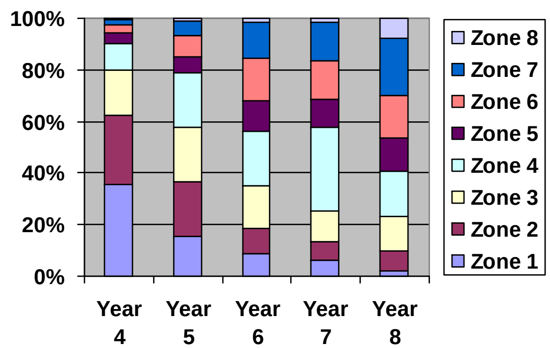

Figure 1. Proportions of students by Zone and Year Level from the initial phase of SNMY Project

The results of the SNMY confirmed the finding of the MYNRP that there was an 8 year range in achievement at each year level and when the LAF Zones were analysed against curriculum expectations, it was evident that up to 40% of Year 7 and 8 students performed below curriculum expectations and at least 25% were well below expected level (Siemon, Breed et al., 2006).

This discrepancy is unacceptable in a country that prides itself on providing opportunities for all (Ministerial Council on Education, Employment, Training & Youth Affairs, 2008). Multiplicative thinking is a key indicator of success in school mathematics in the middle years and as such it is imperative that the key ideas and strategies that underpin the transition from additive to multiplicative thinking are clearly articulated and understood by teachers and curriculum developers. A focus on the big ideas in number is essential to inform more targeted approaches to the teaching and learning of mathematics to ensure that all students have the opportunity to deepen their understanding and participate fully and effectively in school mathematics.

Big Ideas in Number and the Australian Curriculum: Mathematics

Scaffolding student learning is the primary task of teachers of mathematics. However, this cannot be achieved without accurate information about what each student knows already and what might be within the student's grasp with some support from the teacher and/or peers. This not only requires a clear understanding of the key ideas, representations and strategies in school mathematics, how they are connected and how they might be acquired over time, it also requires assessment techniques that expose student thinking, interpretations of what different student responses might mean, and some practical ideas to address the particular learning needs identified (Siemon, 2006). As we have seen above, this is particularly important in relation to a relatively small number of 'big ideas' and strategies in Number.

The *Assessment for Common Misunderstanding* tools [hereinafter referred to as the tools] were developed for the Victorian Department of Education and Early Childhood Development (see Siemon, 2006) to address this need. They draw on research-based tasks and represent what Callingham (2011) has referred to as productive assessment, in that they provide useful, timely, appropriate information fit for purpose. Based on earlier work with pre-service teachers and schools in the Northern Territory (Siemon, Enilane & McCarthy, 2004), the tools were developed to help teachers better "understand and monitor their individual students' developing strategies and particular learning needs" (National Curriculum Board, 2008, p. xiv) in relation to a small number of very big ideas in Number without which student's progress in mathematics will be severely restricted. These ideas are summarised in Table 1. The first five ideas are then considered in terms of their associated tools and the ACM (version 1.2) as it is these ideas that most concern the development of multiplicative thinking in the middle years of schooling.

Table 1. Big ideas in Number by stages of schooling (Siemon, 2006)

By the end of:	Big Idea	Indicated by:
Foundation Year	Trusting the Count	Access to flexible mental objects for the numbers to ten based on part-part-whole knowledge derived from subitising and counting (e.g., know that 7 is 1 more than 6, 1 less than 8, 5 and 2, 2 and 5, 3 and 4 without having to make or count a collection of 7)
Year 2	Place-value	Capacity to recognise and work with place-value units and view larger numbers as counts of these units rather than collections of ones (e.g., able to count forwards and backwards in place-value units)
Year 4	Multiplicative Thinking	Capacity to work flexibly with both the number in each group and the number of groups (e.g., can view 6 eights as 5 eights and 1 more eight). Recognises and works with multiple representations of multiplication and division (e.g., arrays, regions and 'times as many' or 'for each' idea).
Year 6	(Multiplicative) Partitioning	Ability to partition quantities and representations equally using multiplicative reasoning (e.g., a fifth is smaller than a quarter, estimate 1 fifth on this basis then halve and halve remaining part again to represent fifths), recognise that partitioning distributes over previous acts of partitioning and that numbers can be divided to create new numbers
Year 8	Proportional Reasoning	Ability to recognise and work with an extended range of concepts for multiplication and division including rate, ratio, percent, and the 'for each' idea, and work with relationships between relationships
Year 10	Generalising	Capacity to recognise and represent patterns and relationships in multiple ways including symbolic expressions, devise and apply general rules

The following descriptions draw on material written for the DEECD in 2006 and 2011 (Siemon, 2006, 2011a) and Siemon, Beswick, Brady, Clark, Faragher & Warren (2011). They comprise a number of easy to administer, performance-based tasks designed to address a key area of Number at different levels of schooling from Foundations to the end of Year 10. In the associated teaching advice a range of student responses is identified for each task and, for each of these, an interpretation of what the response implies is provided together with targeted teaching suggestions.

Trusting the Count

The term 'trusting the count' was originally proposed by Willis (2002) to draw attention to the fact that children may not believe that if they counted the same collection again they would arrive at the same amount. More recently this term has been appropriated and extended to refer not only to the belief that counting will produce an invariant result (literal interpretation), but also to the capacity to access mental objects for the numbers to ten that render counting unnecessary in most dealings with those numbers (Siemon et al., 2011). Derived primarily from extensive experiences with subitising (the ability to recognise small collections without counting), a child trusts the count for a number such as 8 when he or she can access a repertoire of knowledge items and images for 'eightness' that obviates the need to represent and count collections of 8 in order to work with 8. Viewed in this way, trusting the count also supports a sense of numbers beyond ten, for example, a collection of 16 can be recognised as 1 ten and 6 more without counting on by ones (Siemon, 2006).

Trusting the count is a big idea that builds on and connects early number ideas derived from counting and subitising. In particular, it presumes children are familiar with the number naming sequence and understand what is meant by *more, less* and *the same* in this context. Trusting the count is not about addition or subtraction, although it is a key component of additive thinking. It is about deeply understanding what each of the numbers to ten means and the various ways in which they might be represented in terms of their parts. It is an essential pre-requisite for understanding larger numbers and developing a sense of quantitative reasoning (Smith & Thompson, 2007).

Two tools are used to evaluate children's capacity to trust the count (see Siemon, 2006). The first assesses children's capacity to recognise numbers to 5 without counting and on this basis to recognise the remaining numbers to ten without counting referred to as *conceptual subitising* by Clements and Samara (2007). The second tool is based on a task developed by Steffe and his colleagues in the early 1980s to evaluate children's counting strategies (Steffe, Cobb & von Glasersfeld, 1988). It is used in this context to examine the extent to which children have access to mental objects for the numbers to ten.

The *Australian Curriculum: Mathematics* [ACM] at this level of schooling refers to the 'language and processes of counting' (ACMNA001) and the ability to 'connect number names, numerals and quantities (ACMNA002), 'subitise small collections' (ACMNA003) and 'compare, order and make correspondences between collections' (ACMNA289). While these capacities are necessary to build mental objects for each of the numbers to ten, they are not sufficient. By the end of their first 12 to 18 months of school, children need a deep understanding of the numbers to 10 that goes beyond the language and processes of counting to ensure that when they hear, read, write, or say a number such as 'seven', they can imagine that number in terms of its parts (e.g., as 1 more than 6, 5 and 2, or 3 and 4) and how it relates to other numbers (e.g., as 1 less than 8 or 3 less than 10), without having to make, count or literally 'see' a collection of 7 objects. A deliberate and explicit focus on the development of *part-part-whole knowledge* is needed to ensure that children move beyond the language and processes of counting to develop mental objects for each of the numbers to ten that they can use flexibly without recourse to materials or models.

Place Value

The big idea of place-value in the early years of schooling is that it provides a system of new units based on the notion that '10 of these is 1 of those' that can be used to work with and think about larger whole numbers in efficient and flexible ways. By the end of their third year of school (generally Year 2), most students can count by ones to 100 and beyond, read and write numbers to 1000, orally skip count by twos, fives and tens, and identify place-value parts (e.g., they can say that there are 4 hundreds 6 tens and 8 ones in 468). However, these behaviours do not necessarily mean that children understand place-value as many students still think about or imagine these numbers as collections of ones, and they are unable to rename numbers in terms of their place-value parts (e.g., rename 476 as 47 tens and 6 ones) or work in place-value parts (e.g., name the number 2 tens less than 5308). That is, they do not recognise tens, and hundreds as units within a larger, place-based system of numeration.

Four tools are used to evaluate the extent to which children understand place-value. The first, the Number Naming Tool is based on a task used by Ross (1989) and explores the meanings children attach to 2-digit numerals (e.g., the meaning of 6 and 2 in 26) and the extent to which they can be distracted by regrouping 26 counters into groups of 4 (i.e., they understand ten as a countable unit). The Efficient Counting Tool indicates the extent to which students can use twos, fives or tens as countable units to count large collections more efficiently. The Sequencing Tool explores the strategies students use to locate a 2-digit number on a 0-100 number line and the Renaming and Counting Tool examines student's capacity to name and rename a 3-digit number and count forwards and backwards in place-value parts from a given 4-digit number.

In the ACM, although number and place-value is used as a thread across all year levels there are only four references to place-value in the content descriptions, one at Year 1, one at Year 3 and two at Year 4. The first, "count collections to 100 by partitioning numbers using place value" (ACMNA014), can be accomplished

without recognising tens as units (e.g., 'partitioning' 43 as 40 and 3 does not emphasise ten as a countable unit as 40 is the name for 40 ones). The next two descriptors are concerned with assisting calculations (ACMNA053 and ACMNA073) and the fourth is concerned with the extension of the place-value system "to tenths and hundredths" (ACMNA079). Number sequences and skip counting are variously referred to in Years 1 and 2 (e.g., ACMNA012 and ACMNA026) but counting by twos, threes, fives or tens does not necessarily mean that 2, 3, 5 and 10 are understood as countable units. A count of 3, 6, 9, 12, … or 10, 20, 30, 40, … could simply be seen as a shortened form of counting by ones.

The ability to "recognise, model, read, write, and order numbers to at least 100" (ACMNA013), "to at least 1000" (ACMNA027), "to at least 10 000 (ACMNA052), and "to at least tens of thousands" (ACMNA072) are necessary pre-requisites for working with larger numbers but again, these capacities do not necessarily mean that children understand the structural basis of the base ten system of numeration or recognise tens, hundreds, and thousands as abstract composite units (Siemon et al., 2011).

Additive thinking is not regarded as a big idea in its own right as it builds upon the two ideas of trusting the count and place value (Siemon et al., 2011). It is evident when children work with numbers as mental objects and rename numbers as necessary to facilitate calculations. For example, asked to calculate 36 and 27, an accomplished additive thinker might draw on her knowledge of place value to recognise this sum as 5 tens and 13 ones and therefore 63. Alternatively, she might add 2 tens to 36 to get 56 then, recognising 7 as 4 and 3, add 4 to 60 then 3 more to arrive at 63.

Multiplicative Thinking

For the purposes of the SNMY project, multiplicative thinking was described in terms of:

- a capacity to work flexibly and efficiently with an extended range of numbers (and the relationships between them);
- an ability to recognise and solve a range of problems involving multiplication and/or division including direct and indirect proportion; and
- the means to communicate this effectively in a variety of ways (e.g., words, diagrams, symbolic expressions, and written algorithms) (Siemon, Breed et al., 2006)

Multiplicative thinking is a critically important 'big idea' as it underpins virtually all of the work in number and algebra in the middle years of schooling. By the end of Year 4 students need to be able to think about multiplication in a number of different ways so they can recognise when multiplication is required and how it relates to division, develop efficient mental strategies and meaningful forms of written computation to solve a wider range of problems, and make connections to fraction representations, percent, rate and ratio. To achieve this they need to experience multiplication and division in ways that support a critical shift in thinking from a reliance on equal groups and repeated addition to a more general understanding of multiplication and division in terms of factor-factor-product (Siemon et al., 2011).

Six tools are provided to evaluate multiplicative thinking at this level. These are summarised in Table 2.

Table 2. Tools used to examine the emergence of multiplicative thinking (Siemon, 2006)

Tool	Designed to evaluate student's capacity to:
Additive strategies	access to mental objects for the numbers to ten and efficient mental strategies for addition and subtraction
Countable units	recognise numbers as abstract composite wholes (Killion, Steffe & Stanic, 1989), that is, as countable units in the absence of physical materials/models
Sharing	share equally, recognise commutativity (e.g., that 3 groups of 4 is the same as 4 groups of 3), appreciate the meaning of 'times as many as'
Array and region	use the properties of arrays and regions to determine the total amount without counting by ones or skip counting
Cartesian product	solve problems involving the Cartesian product or 'for each' idea of multiplication (e.g., the total number of lunch orders given three types of bread, 4 different fillings and 2 types of fruit)
Simple proportional reasoning	Use 'if … then' reasoning to solve simple proportional reasoning problems (Clarke & Kamii, 1996)

In the ACM, the only reference to any of these key ideas is in Foundations where sharing is mentioned (ACMNA004) and in Year 2 where students are expected to recognise and represent "multiplication as repeated addition, groups and arrays" (ACMNA031) and "division as grouping into equal sets" (ACMNA032). However, sharing a collection equally does not necessarily indicate multiplicative thinking unless students recognise the relationship between the dividend and the quotient (Nunes & Bryant, 1996) and working with arrays is no guarantee of multiplicative thinking either unless the focus of attention is shifted from a count of groups of the same size (additive) to a given number of groups of any size (Siemon et al., 2011). Importantly, the region idea is not mentioned at all and yet this underpins the 'area' or 'by' idea of multiplication (i.e., each part multiplied by every other part) which is needed to support the multiplication of larger whole numbers (e.g., 2-digit by 2-

digit multiplication), the interpretation of fraction diagrams (e.g., thirds by fifths are fifteenths) and ultimately the multiplication of linear factors.

In Year 3 students are expected to "recall multiplication facts of two, three, five and ten and related division facts" (ACMNA056). This wording together with the reference to number sequences "increasing and decreasing by twos, threes, fives and tens" (AMNA026) in Year 2 implies that the multiplication facts are learnt in sequence (e.g., 1 three, 2 threes, 3 threes, 4 threes, 5 threes etc) rather than on the basis of number of groups irrespective of size (e.g., 3 of anything is double the group and one more group). The references to "investigate number sequences involving multiples of 3, 4, 6, 7, 8, and 9" (ACMNA074) and "recalling multiplication tables" (Fluency proficiency) at Year 4 reinforce this observation. This is unfortunate given the reported success of alternate approaches to learning the multiplication facts based on commutativity and distributivity (not mentioned in the ACM until Year 7) and renaming numbers (e.g., McIntosh & Dole, 2004; Siemon et al., 2011).

Factors and multiples are referred to in Year 5 (ACMNA098), "properties of primes, composite, square and triangular numbers" in Year 6 (ACMNA122), indices in Years 7 and 8 (ACMNA149 & ACMNA182), and solving problems involving specified numbers and operations across year levels (e.g., ACMNA100, ACMNA101 and ACMNA103). These content descriptors vary little from the topic-based curriculum of 50 years ago. There is no suggestion of the connections between them or that something other than a repeated addition model of multiplication is needed to support a deep understanding of factors and indices (Confrey, Maloney, Nguyen, Mojica & Myers, 2009).

Partitioning

The idea that a collection or a quantity can be expressed in terms of its parts is fundamental to developing a strong sense of number. This can be done additively (as in part-part-whole knowledge and renaming whole numbers in terms of their place-value parts) or multiplicatively (as in the production of equal parts). To clarify this distinction, Confrey and her colleagues (Confrey, et al., 2009) introduced the term equipartitioning (or splitting) to refer to

> behaviors that create *equal-sized groups*. In addition, we would assert that division is most directly derived from equipartitioning, with multiplication following as its inverse, rather than the traditional view that multiplication precedes division. … Equipartitioning/splitting as an operation leads to partitive division as well as to multiplication. (p. 347)

Multiplicative partitioning, equipartitioning, or partitioning as it is used in this context is a 'big idea' that underpins the capacity to work meaningfully with rational numbers and their representations. In particular, to compare, order and rename fractions, build strong connections between multiplication, division, fractions and decimals, and support the extension of multiplicative thinking to rate, ratio and percent (Siemon et al., 2011).

Seven tools examine the following indicators of partitioning as it is used in this context.

- Distinguish between fraction and non-fraction representations in multiple settings.
- Recognise the relationship between the number of equal parts and the size and name of the parts (e.g., as the number of parts/shares increase the size of each part or share decreases).
- Use efficient multiplicative strategies to construct indicative fraction diagrams and line models, name and record common fractions and decimals
- Recognise that the relative magnitude of a fraction depends upon the relationship between the numerator ('how many') and the denominator ('how much').
- Use meaningful strategies to compare, order and rename common fractions and decimal fractions.

In the early years, the ACM refers to the capacity to "recognise and describe half as one of two equal pieces" (ACMNA016) and "to recognise and interpret common uses of halves, quarters and eighths of shapes and collections" (ACMNA033) but no mention is made of the important link to sharing which provides a powerful basis for the creation of equal parts and the link between fractions and partitive division (Nunes & Bryant, 1996). In Year 3, students are expected to be able to "model and represent unit fractions including 1/2, 1/4, 1/3, 1/5 and their multiples to a complete whole" (ACMNA058). This suggests that fraction symbols are expected at this stage, which is problematic given the well known difficulties associated with interpreting fraction symbols and representations (e.g., Lamon, 1999). Also, the reference to counting fractions at Year 4 (ACMNA078) appears to privilege the fraction as number or measure idea over the many other representations of fractions, for example, part-whole relations, quotients, ratios and operators (Confrey et al., 2009; Lamon 1999). Focussing on fractions as measures could also lead to an over-reliance on additive, whole number-based approaches to locating fractions on a number line at the expense of multiplicative approaches such as partitioning.

The inclusion of hundredths at Year 4 (ACMNA079) is mystifying in view of the research on decimal fraction misconceptions (e.g., Steinle & Stacey, 2004). It has possibly been included here because calculations to the nearest cent have been included at this level (ACMNA080) but there is little/no evidence to suggest that being able to work with money contributes to a deep understanding of decimal fractions. The fact that there is no mention of percentage benchmarks such as 50%,

25% at this level is also somewhat surprising given children's capacity work with repeated acts of halving (percentages are not referred to at all until Year 7).

Comparing and ordering "common unit fractions" and locating them on a number line (ACMNA102) is consistent with partitioning as is recognising "that the number system can be extended beyond hundredths" (ACMNA104). However, given that partitioning supports generalisations about how fractions might be renamed (e.g., if the total number of parts are increased by a certain factor then the number of parts required is also increased by that factor), it seems strange that the comparison of unlike fractions (fractions with unrelated denominators) has been pushed back to Year 7.

Proportional Reasoning

Proportional reasoning involves recognising and working with relationships between relationships (i.e., ratios) in different contexts. Proportional reasoning is important as it underpins the work done in other domains of mathematics (e.g. scale diagrams, the analysis of similar figures in geometry, and calculations involving percentages in financial mathematics) and provides a powerful basis for understanding functional relationships more generally.

The following indicators of proportional reasoning are examined by eight tools.

- Use relational thinking (multiplicative) as opposed to absolute thinking (additive) to analyse change over time or compare relationships.
- Identify and describe relationships between quantities in a range of problem contexts
- Work flexibly and confidently with the quantities involved (i.e., measures, rates and/or ratios expressed in terms of natural numbers, rational numbers, percents and/or integers).
- Use a scale factor to enlarge/reduce a 2-dimensional shape or estimate distances on a scale map.

The ACM does not refer to proportional reasoning explicitly until Year 9 where reference is made to solving problems involving direct proportion and simple rates (ACMNA208) and enlargements, similarity, ratios and scale factors in relation to geometrical reasoning (ACMMG220 & ACMMG221). While many of the prerequisite skills are included in Years 6 to 8, these appear in the form of disconnected and only slightly differentiated skills. For example, "find a simple fraction of a quantity" (ACMNA127) at Year 6, "express one quantity as a fraction of another", "find percentages of quantities and express one quantity as a percentage of another" (ACMNA 155, & ACMNA158) at Year 7, and solve a range of problems involving percentages, rates and ratios (ACMNA187 & ACMNA188) at Year 8.

Importantly, there is nothing to suggest how these skills relate to one another or their rich connections to multiplicative thinking more generally.

The Verdict

While it is still early days and it remains to be seen how educational systems and schools will work to bring the ACM to fruition, the casual reader could be forgiven for thinking that the ACM is just a thinner version of existing State and Territory mathematics curricula. The big ideas described here are not entirely absent from the ACM but they are not visible in ways that might provide "a powerful transformational force for deepening teacher knowledge for teaching mathematics and energising practice over time" (Siemon, 2011b, p. 68).

The proficiencies which value conceptual understanding alongside procedural fluency as well as mathematical reasoning and problem solving, offer some potential to 'connect the dots' but the description of these at each year level is very brief and it will require a significant commitment to teacher professional learning to achieve this. Recent experience points to the benefits of focussing on the big ideas and using these as an organising frame to target teaching to learning needs and improve student outcomes. Some of this experience is reported in what follows.

Working with the Big Ideas in Number

The following cases illustrate how the *Assessment for Common Misunderstandings* materials have been used in South Australia and Tasmania to promote teacher professional learning and inform teaching practice. The teacher's names have been used with permission and their quotes are included in italics to distinguish the teacher voice from other quotes included in the chapter.

The South Australian Experience

In August 2009 the South Australian Department of Education and Children's Services [DECS] introduced the Literacy and Numeracy National Partnership [LNNP] project which placed 14 Numeracy Coaches in primary schools. This initiative was funded through the Australian Government's Smarter Schools National Partnership. The Numeracy Coaches are each based in one or two schools and work with teachers and school leaders to improve numeracy outcomes across the school. Coaches are supported by an intensive professional learning program focussing predominantly on the skills of coaching, mathematical pedagogical content knowledge, working with student achievement data, and whole school improvement.

The *Assessment for Common Misunderstanding* materials were used with permission from DEECD as the basis for the coaching initiative and coaches were provided with a hard copy of these materials which were referred to as the *Big Ideas in Number*. Extensive professional learning on the big ideas was provided to the numeracy coaches who then worked in their schools with teachers to implement these ideas and strategies in their classrooms. As part of the LNNP evaluation, coaches were interviewed about the impact and outcomes of the project. The following examples are illustrative of their feedback.

Brianna is the LNNP Numeracy Coach at a rural primary school and she has been working closely with Emily and her Reception/Year 1 (R/1) class on the big idea of Trusting the Count. Previously Emily had a strong focus on children being able to count and match number symbols to collections and number names. She now recognises that this was important but not enough. She believes that the increased emphasis on developing children's deep understandings of the numbers 0 to 9 and particularly their part-part-whole understanding (i.e., that seven is five and two, three and four and so on), has paid dividends as children can work flexibly with numbers from the earliest years. Her children don't just learn what seven is but also how to break seven into its parts and put them back together again!

In this class Emily and Brianna have used a wide range of materials, such as subitising cards, ten frames, dice, and clothes lines to develop understanding. They have also explored electronic technologies like Bee Bots, Interactive whiteboards and iPods to support learning. The children program Bee Bots to move a given number of steps along a number line, guessing where it will end up or matching the numeral to the number line position. They talk about 'how many more steps to ten'. They develop their understanding of doubles by playing a doubles dice game on the interactive white board. In this game a die is 'rolled' and the outcome displayed on the board. The children have to double the number rolled and then select the answer from a line of numbers from 1 to 12. They get one point for each correct answer and have a time limit of 60 seconds to get the highest score they can. Emily then records this score for each child to map their improvement over time. Some of the children can double and find the answer in the line up so quickly that visiting adults can't match the student's score. The iPod Touches have been a really big hit with the children. They have quickly mastered the navigation and use of the visual menus. Emily and Brianna have found some age appropriate iPod apps that focus on early number skills and they continue to look for more.

Over the time that Emily has been working with the big ideas in number she has learnt a lot about how children learn number concepts. She expresses her own learning as,

> I am able to make the learning more hands on and by watching children engaged in learning activities I can often 'see' what they are thinking. I have a better grasp of what I'm looking for and my own understanding of conceptual development in the number strand continues to grow. I can now target children's learning more accurately to their needs so there is less maths time wasted.

Madeline, also working with numeracy coach Brianna, found the big ideas really helped her to change her approach to teaching multiplication facts to her Year 3/4 class. She now emphasises a strategies approach rather than relying on rote learning – the students know, for example, that 5 times a number is 'half of ten of it' or that to find three times a number they can double the number then add on the original number. Madeline has talked with many of the parents to explain this new approach and says that once she explains the thinking behind a strategies approach to the multiplication facts and the benefits for student learning the parents are very supportive. This focus on strategies in the learning of multiplication facts is consistent with the way that Madeline now encourages students to explain their thinking in maths more generally. For example, her class do a short subitising activity each day and then explain how they partitioned the numbers to find the total. Depending on the way that the objects are arranged students might see 8 objects as 'a three and a five' or 'two fours' or 'two threes and a two'. Students share their strategies and soon appreciate that numbers are not fixed, they can be broken apart and put back together again and that there are lots of ways to do this. This understanding flows into strategies for mental addition so that for example, 18 + 27 could be 18 + 30 – 3 or 18 + 2 + 25 depending on which mental image is more powerful for the individual student.

Madeline uses the Big Ideas in Number diagnostic tools regularly to identify gaps in student learning. Previously, she says, some children were able to 'hide' or bluff their way along – they appeared to be learning or at least they didn't stand out as not learning. When using one of the diagnostic tools however students cannot bluff their way through, she knows what they do or do not understand. Madeline finds the Advice section particularly helpful to ensure that she focuses new learning appropriately for each student.

Madeline has reflected back on her introduction to the Big Ideas and says that she initially found the new language, terms like subitising, renaming, trusting the count, quite daunting but with Brianna's help she persevered. She now finds it easier to have discussions with colleagues because if they talk about 'renaming' for example they all know what is meant. The students too have enjoyed playing around with new language. For example, when working on place value the students really enjoyed playing with the language and using made up names like 'onety-one, onety-two' and so on.

As an early career teacher Madeline has found working on the Big Ideas in Number with the support of a Numeracy Coach to be a huge boost to her confidence as a teacher of mathematics. In her first year of teaching in 2009 she worried that her lack of confidence with maths would spill over into the learning of her students. Now she says,

> I'm so glad I have worked with the Big Ideas in Number as a young teacher and that it is now becoming second nature. When I started teaching I moved on too quickly without really building strong foundations of understanding. I can now look at the child not just the curriculum. I can target learning to my students' needs not just teach Year 3 or Year 4. I started off dreading maths but now I enjoy teaching maths more than anything.

Guy is an experienced teacher in a metropolitan primary school with a Year 5/6 class. Guy and Chris, the Numeracy Coach, have been working together on the Big Ideas since late 2009 and over this period they have used the diagnostic tools and advice and introduced the students to a range of learning games and activities designed to build on students' immediate learning needs.

Over time they have developed many other activities that have spring-boarded from the original advice. One of these is using the additive strategies cards (see Figure 1 below) that are projected on the smart-board.

33	18
22	?

?	12
12	48

Figure 1. Additive strategies cards (adapted from Siemon, 2006)

Students record their answers and the way they worked it out. They then share their strategies (e.g., doubling, near doubles, make to the nearest ten, number splitting and compensating) and discuss the relative efficiency of each. Teachers and students invent new and interesting variations of these 4 square problems and as students become more efficient the problems become more difficult!

Since working with the big ideas Guy has placed more emphasis on children talking about their maths learning and recording their thinking in as many ways as possible. This approach helps develop mathematical language but also as students learn about successful approaches from their peers they increase their flexibility in working with number. He has implemented a range of classroom strategies and protocols to support this emphasis. One example is the use of a laminated A3 sheet on which the students record their thinking in words, symbols or diagrams. As this work is easily changed or erased students are more inclined to write and record than they are on paper or in a maths book. The students can hold up their laminated sheets to help explain their thinking to other students and discuss the accuracy and efficiency of their strategies.

Chris and Guy have used some of the Big Ideas in Number diagnostic tools as whole class or group activities. For example, an activity they use is called Thinking Strings where a student or teacher randomly places a line of magnetic base 10 blocks on the white board as shown. Students then count on in place value parts and record their thinking as shown in Figure 2.

Figure 2. Use of thinking strings and base 10 blocks

When students are involved in these activities Chris and Guy have opportunities to sit with individual students, observe their responses, probe their thinking and give immediate constructive feedback.

In working with the language of the *Big Ideas in Number*, Guy and Chris have become aware that for many students talking about mathematics strategies and learning is difficult. These students need explicit scaffolding and support to develop their mathematical vocabulary and the confidence to use it in front of their classmates.

The Tasmanian Experience

Over the past two years interest in improving student outcomes in mathematics has grown in many Tasmanian schools. Professional learning focusing on the big ideas in number (Siemon, 2006) was seen by the Tasmanian Department of Education as a means of focusing teachers' attention on what is important and to encourage school communities to recognise the importance of developing strong foundations in number for all strands of the mathematics curriculum.

Teachers and school mathematics leaders from all parts of the state have participated in professional development workshops and there has been a very high level of take up of the *Assessment for Common Misunderstandings* materials sourced and used with permission from DEECD.

Many teachers have been surprised about the misunderstandings their students have and they have realised the gaps in student understanding which have contributed to poor performance in NAPLAN and other assessments. For example, Maree who teaches a grade 5/6 class realised that some of her students could not subitise small collections (ACMNA003) and solved all numerical problems by counting by ones. This gave her valuable information about where to target her teaching with this group of students. Using a flip camera she was able to capture some students while they were assessed using the Trust the Count assessment tool and the video footage has been used in several professional learning sessions to help other teachers unpack the assessment tools and to realise that they too may have students who struggle with ideas well below where they might expect them to be in an upper primary class.

Amanda who teaches grade 5/6 found that she had students who struggled with early ideas in place value. She used the place value tools to probe student understanding and the advice from the website to differentiate instruction to address the student's learning needs. Teaching groups were formed to target intervention and pre and post testing showed significant gains in student understanding.

Similarly, school mathematics leaders Pam and Sylvia from a large primary school note that:

> The big ideas provide our teachers with a focus on the important ideas in mathematics. The big ideas leave no doubt about what is sequentially important for students to know and understand. They stress the importance of building mental images and visualising concepts. The focus is on hands-on learning that promotes deep understanding of maths concepts and on building the language of maths. The Assessment for Common Misunderstandings tools are being developed into kits and will support teachers in assessing students but will also provide teachers with a "where to next?' for planning for learning. The big ideas will underpin our whole school approach document in conjunction with the Australian Curriculum.

With the decision to fully implement the ACM in 2012 in all Tasmanian Schools it became obvious that teachers need good pedagogical content knowledge and access to assessment tools which support their curriculum decision-making in response to student learning needs.

> Curriculum-related assessment information is required for a detailed analysis of students' learning needs. These kinds of data are more useful for the purposes of diagnosing students' learning needs than assessments focused more on identifying normative achievement, but not related to the curriculum (Timperley, 2009, p. 22)

As a consequence, in 2011, 11 schools in one geographic region of the state used the big ideas in number and data derived from the associated tools to focus teacher professional learning on meeting student learning needs. The best evidence synthesis of effective professional learning for teachers (Timperley, 2007) was used as a foundation for the project. Teachers adopted an inquiry approach to their teaching and professional learning based on identified student needs and data, not generic professional learning based on what the Tasmanian Department of Education might *think* teachers need. This approach was framed by an adaptation of Timperley's (2009) teacher inquiry and knowledge-building cycle (see Figure 3) on the grounds that while high quality assessment is valuable, "much more is needed to improve teaching practice in ways that have a substantive impact on student learning" (p. 21).

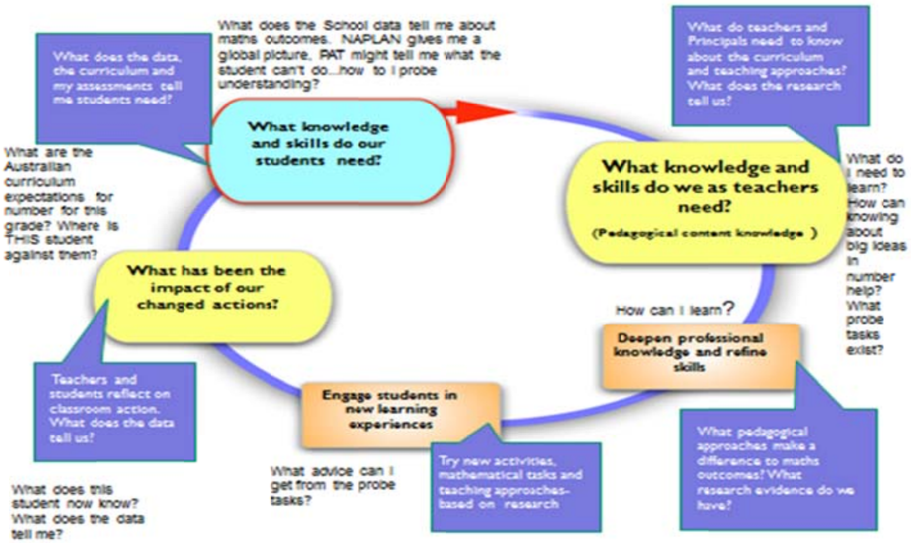

Figure 3. Teacher inquiry and knowledge building cycle (adapted from Timperley, 2009)

Project schools were provided with a full set of the tools and teachers involved in the project worked with school-based lead teachers and external facilitators to engage in data-driven conversations to determine professional learning needs and teaching focus. The big ideas framework and associated assessment tools enabled teachers to go beyond the data they may have gathered from other sources (e.g., NAPLAN) to delve into student misconceptions in number and use the advice to teach in richer, more focussed and intentional ways (e.g., see Figure 4).

Figure 4. Children working with ten-frames and bead strings.

This work will be invaluable in building teacher capacity to focus assessment and teach for the depth and understanding which is a key pedagogical underpinning of the mathematics curriculum, for example, "it is preferable for students to study fewer aspects in more depth rather than studying more aspects superficially" (National Curriculum Board, 2009, p. 14). This will also help teachers become more aware of the finer detail of the key ideas underpinning the content descriptors in the ACM which are often very broad and open to interpretation. As stated in the *Shape of the Australian Curriculum: Mathematics* (National Curriculum Board, 2009), "teachers can make informed classroom decisions interactively if they are aware of the development of key ideas" (p. 14).

John, who is a facilitator/coach in the project and a former secondary mathematics teacher, confirms this view when he says:

> the big ideas have focused my thinking around the sequential development of ideas. To address the diverse ability range of our students, teachers need to understand how mathematical concepts are developed and teachers need to be supported to translate this into improved classroom practice.

His colleague Wendy, with a background in Early Childhood education concurs when she reflects on how her work has altered since she has focused on big ideas in number:

> Now that I have a deeper understanding of [the] big ideas it has made me more aware of the possible misunderstandings that children can have in key areas of number. A huge change in my thinking was realising that as an ECE classroom teacher that I hadn't been taking children back far enough when designing intervention programs. It was a struggle to get them to learn and apply strategies and when faced with solving problems they would always count on in ones.
>
> This has impacted greatly on my messages to ECE teachers now:
>
> - Spend more time developing mental images especially with subitising tasks (it is crucial...then hopefully we won't have children in Grade 6 and beyond still counting on in ones!)
> - Teachers of Grade 1 and Grade 2 should spend more time on and give children more opportunities for developing Place Value concepts...Counting collections and recording the number, bundling etc

As teachers involved in this project further explore the ACM and its focus on the four proficiencies, there is potential for re-visiting the big ideas in number in new and exciting ways, emphasising the explicit teaching focus for teachers and the importance of tasks that are selected to focus on both content descriptors and proficiencies.

Tasmanian schools have seen the potential of teacher professional learning based on in-depth knowledge of student understanding of key ideas in number. The big ideas in number provide a valuable framework for exploring questions such as "what is important?" "What will give us the greatest leverage in improving student outcomes?" Indeed, these big ideas are influencing our small state and engaging teachers in new learning for themselves and their students!

Conclusion

This chapter considered why big ideas have become a focus of attention in recent years in relation to the teaching and learning of mathematics and the design of school mathematics curriculum. It was suggested that this is a response to evidence from international assessments and large-scale numeracy research projects that many students in the middle years lack the depth of knowledge needed to critically apply mathematics. A focus on 'big ideas' and the links between them is needed to highlight key ideas and strategies at different levels of schooling, thin-out the overcrowded curriculum, and help deepen teacher knowledge and confidence to support more targeted teaching approaches. This is particularly the case for Number which has been shown to be the area most responsible for the range in mathematics achievement in the middle years.

Not everyone will agree with the notion of big ideas presented here, that is, as important organizing frames for thinking about and working with mathematics without which student progress in mathematics will be seriously impacted. This view motivated the choice of the big ideas in number used to inform the design of the *Assessment for Common Misunderstandings* materials, five of which are considered in the chapter, namely, trusting the count, place-value, multiplicative thinking, partitioning, and proportional reasoning. While these provided a useful lens to examine the ACM it would be naïve to think that a national mathematics curriculum could be organised in terms of these big ideas. Curricula serve many purposes and have many audiences but it is not unreasonable to suggest that in implementing the ACM and thinking about the type of professional learning needed to support a 21st century mathematics curriculum, serious consideration be given to these overarching themes and how these relate to and help connect the many seemingly disjointed behaviours that inevitably have to be listed in a document such as the ACM.

The South Australian and Tasmanian experiences of using the tools to inform and better target teaching practice indicate that a focus on the big ideas in Number 'works'. 'It's not rocket science' - teacher feedback on the use of the tools report significant improvements in student engagement and progress where student learning needs in relation to a small number of 'really big ideas' in Number are more accurately identified and the teaching is more closely targeted to meeting those needs.

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Chapter 3
The Role of Algebra and Early Algebraic Reasoning in the Australian Curriculum: Mathematics

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In this chapter we describe the place of algebra (Number and Algebra [NA] content strand) and the development of early algebraic reasoning in *The Australian Curriculum: Mathematics*). We raise critical questions about the scope, depth and sequence of the algebra (NA) strand and the interrelationships between the Proficiencies and Number and Algebra. This is informed by a review of the research bases for prioritising algebra within mathematics curriculum reform generally. We describe the potential affordances and limitations of algebra for mathematics teaching and learning from preschool through secondary grades. The challenges facing teachers and the impact on the development of teachers' pedagogical content knowledge are discussed. Implications for further research on the implementation of the algebra strand within the Australian context are outlined.

Introduction

Algebra is considered an integral part of mathematics curriculum. In recent decades the view that algebra should be introduced in the primary grades has been more widely accepted with even more recent emphasis on patterning and early algebraic reasoning in the preschool and early grades. Early algebraic thinking develops through an awareness of the structural relationships of patterns and later in the structure of arithmetic (Carraher, Schliemann, Brizuela & Earnest, 2006; Kaput, 2008; Mason, Stephens, & Watson, 2009). Patterning in the early years often does not involve numbers but can still lead to simple forms of generalisation. Mason, Graham, and Johnston-Wilder (2005) argue for a focus on generalisation in early mathematics learning. Early algebraic reasoning can develop from a natural awareness of generalisation and ability to express generality. Thus the role of algebraic thinking in the curriculum is about promoting generalisation in mathematics and reflecting this in a scope and sequence throughout the primary

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and secondary years. Moreover Warren asserts that early algebraic thinking refers not only to thinking about algebra early, but is also about re-looking at number from a more structural perspective (Warren, personal communication, September, 2011).

Research Bases for Prioritising Algebra within Mathematics Curriculum Reform

In past decades research on the teaching and learning of algebra has largely focused attention on secondary students' difficulties. However, the problems associated with the teaching and learning of algebra can be traced to the development, or lack thereof, of early algebraic reasoning right back to the prior-to-school context. Increasing interest in research about algebra in the primary school has ensued in order to investigate students' understanding of arithmetic based on mathematical generalisations and understanding of basic algebraic principles (Davis, 1985). More broadly, there is now a growing body of research on the development of early algebraic reasoning in the pre-primary school years (Blanton & Kaput, 2005; Carraher & Schliemann, 2007; Warren & Cooper, 2008; Yeap & Kaur, 2008) and prior to formal schooling (Papic, Mulligan & Mitchelmore, 2011).

There is also recognition in the research that arithmetic and algebraic thinking can and should be intertwined in the early years with each supporting the other. Thus it is crucial that algebraic thinking is developed in parallel with arithmetic thinking from an early age. Not only does this result in a deeper understanding of our number system but also forms a strong basis on which to build formal algebraic thinking in the later years. Generalisations from patterning involve inductive reasoning which research has shown that young students are capable of. This has implications for the curriculum and the way we teach arithmetic in primary contexts. One is that patterning and generalisations can only be reached from a range of examples in both numerical and non-numerical contexts. Thus the focus moves away from answering particular problems to exploring an array of examples of that particular problem type – with a focus on the structural aspects (Warren, personal communication, September, 2011).

Learning frameworks and programs and aligned professional development initiatives are now focusing on patterning and structural relationships in mathematics, including equivalence, growing patterns and functional thinking (Mulligan & Mitchelmore, 2009; Mulligan, English, Mitchelmore, Welsby & Crevensten, 2011; Warren & Cooper, 2008). These initiatives integrate important aspects of counting and arithmetic, common to numeracy programs but focus attention on common underlying processes that develop mathematical generalisation.

Current studies are providing increasing evidence that early algebraic thinking develops from the ability to see and represent patterns and relationships. Papic et al. (2011) found that preschoolers' ability to recognise the structure of a simple pattern is central to the notion of unit of repeat and the development of composite units necessary for understanding multiplication. Other research shows strong interrelationships between spatial structuring, patterning and number concepts (van Nes & de Lange, 2007) and the relationship between analogical reasoning and patterning (English, 2004).

New research and cross-cultural comparisons of mathematics curricula and achievement indicate that children in the 4–8 year age range can develop many concepts and processes described as typical of students from 8–12 years of age (i.e. equivalence, functions, complex repetition, multiplicative thinking and proportional reasoning). There is strong current and past research evidence that children in the K–2 years and prior to formal schooling are able to use models, pictures, and symbols to represent ideas, and justify and form simple generalisations. Thus, a new Australian curriculum presents opportunities to develop early algebraic reasoning through the Number and Algebra strand based on growing research evidence with younger students that was not available even a decade ago.

The Role of Algebra in Mathematics Curriculum

Over two decades ago, attempts to provide coherent goals and content for Australian mathematics curriculum resulted in *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1990). A content strand was exclusively devoted to algebra and, in turn, influenced the development of algebra in state-based curriculum reviews. Bands A/B descriptions highlighted the importance of developing algebraic thinking during the primary years and asserted that the notions of generality, variation and function and unknown quantity are critical, not only to algebra but are implicit in the content of other strands of mathematics.

Most state-based Australian mathematics syllabi K-6 implemented in the past decade or more have included algebra as a strand of mathematical content or as central to working mathematically or problem solving. Patterning has been often paired as 'patterns and algebra' and usually linked to the number strand, and perhaps chance and data exploration. For example, in the current NSW Mathematics K-6 syllabus (Board of Studies NSW, 2002) the patterns and algebra strand is linked to the structure of arithmetic with the intent that students will develop knowledge, skills and understanding in patterning, generalisation and algebraic reasoning. Similarly in the Victorian mathematics curriculum algebra forms a central thread of the "structure" strand. The idea that algebraic reasoning and generalisation can be

developed throughout the K-6 curriculum has been espoused but key ideas regarding structural relationships such as equivalence and function have not been developed from Kindergarten/Prep as a coherent sequence. Early algebraic reasoning, with the intent to encourage generalisation through mathematical patterns, relationships and the structure of arithmetic is not really expected until late in the primary and early secondary grades.

Another issue is that even though some aspects of algebra may be visible, perhaps the necessary pedagogical content knowledge is not well formed by many teachers and thus implementation has been limited (Asquith, Stephens, Knuth, & Alibali, 2007; Ball, Thames & Phelps, 2008).

The Place of Algebra (Number and Algebra [NA] Content Strand) in the Australian Curriculum: Mathematics

The *Shape of the Australian Curriculum: Mathematics* paper (Australian Curriculum, Assessment and Reporting Authority, 2009) views the new curriculum as an opportunity for all students to access the study of mathematics. "The study of algebra clearly lays the foundations not only for specialised mathematics study but for vocational aspects of numeracy" (p. 11). Further, participation in algebra, for example, is connected to finishing high school and participation in the workforce. However, students' difficulties in studying algebra during the compulsory years often contribute to poor achievement and low retention rates in mathematics.

The Scope, Depth and Sequence of the Patterns and Algebra Sub-strand

Algebra is combined with number as the Number and Algebra strand, with the algebra content organised into two sub-strands: Patterns and Algebra (Foundation to Year 10) and Linear and Non-linear Relationships (Year 7 to Year 10).

Number and Algebra content strands are paired, as each enriches the study of the other. The underlying principle is that generalising arithmetic operations needs to take place in the primary school years in order to foster a deeper understanding of number and number operations, and to provide a bridge to the more formal study of algebra in the junior high school (Jacobs, Franke, Carpenter, Levi, & Battey (2007). Research in number properties and operations articulates the important connection between general structural principles underlying numerical relationships (Mason et al., 2009). For example, $a - b + b = a$ where the numbers operate as quasi-variables since the numerical relationships hold for all values and can be generalised (Fujii & Stephens, 2008). Missing number sentences are another example of structural numerical relationships that can be expressed algebraically, for example, $\Delta + 23 = 19 + 34$ or $26 \times 3.5 = \Delta \times 10$. "A deep understanding of equivalence and compensation is at the heart of structural thinking and arithmetic"

(Mason et al., 2009, p. 23). This provides a foundation for understanding equivalence relationships that underpin algebraic thinking. For younger children the same notion is evident in equivalent stories (Warren & Cooper, 2009) or simple arithmetic word problems where the structure of the problem may be a 'missing addend' or 'start unknown' situations. Blanton and Kaput refer to 'algebrafying arithmetic' where generalising arithmetic structures is a way of developing algebraic reasoning (Blanton & Kaput, 2003). Viewing simple functional relationships using a table of values for example, allows the numerical relationships to be expressed as pattern and as a generality whether it is symbolised algebraically or not.

> Students apply number sense and strategies for counting and representing numbers. They explore the magnitude and properties of numbers. They apply a range of strategies for computation and understand the connections between operations. They recognise patterns and understand the concepts of variable and function. They build on their understanding of the number system to describe relationships and formulate generalisations. They recognise equivalence and solve equations and inequalities. They apply their number and algebra skills to conduct investigations, solve problems and communicate their reasoning (ACARA, 2011, p. 2).

Although the aim is to build strong interrelationships between Number and Algebra, and the curriculum does provide a more integrated treatment of number and algebra, the content elaborations do not sufficiently emphasise the structural relationships. For example, the sub-strand Patterns and Algebra is really limited in the early years to natural patterns, simple repetition with objects and numbers, and skip counting. This restricts the curriculum from the very foundations to a traditional concept of pattern as simple repetition, rather than providing a more coherent overview of the important interrelationships between number and algebra, such as equivalence. In the secondary years (7 -10) although the concept of a variable is covered is some depth in Year 7 the focus shifts to simplifying and manipulating symbolic expressions such as procedures for factorisation and expansion. Explicit links back to number properties as the basis for developing algebraic relationships are not provided as guidance for teachers.

> A more integrated treatment of number and algebra, rather than just 'pairing' the sub-strands, provides opportunities for teachers to plan learning in a connected way. This allows students to understand more deeply number and number operations, especially in the middle and upper primary years. Similarly a more unified treatment of number and algebra can assist students to make a smoother transition to formal algebra in the secondary school (Stephens, personal communication, Sept, 2011).

Number and algebra might have also been paired with measurement or statistics and probability. The broader view of algebraic thinking might have also been linked to spatial aspects or graphical representations. Further, the interrelationship

between concepts proposed in the rationale seems to be contradicted in the content strand diagram that bears no relationship to interconnectedness between, for example, statistics, number, and algebra. Further, the diagram appears to indicate that there is a hierarchy of concepts beginning with number and algebra, and becoming increasingly layered to include more sophisticated concepts. The explicit role of algebraic reasoning in the Proficiencies strand in the context of problem solving is difficult to envisage.

Patterns and Algebra from Foundation to Year 2

Although the intent of the Foundation to Year 2 section is to describe the basic foundations for learning mathematics, much of the content described is preserving the status quo of curriculum content typical of Australian states. The notion that mathematical learning begins at five years of age (foundations) does not take into account the rich development of mathematical thinking that occurs prior to, and in the first three years of formal schooling, including the notion that young children are capable of abstract ideas and simple generalisations that can promote algebraic reasoning.

The scope of the number and algebra descriptors is limited and does not reflect current research with 4 to 6 year-olds that shows a significant number of children able to move beyond these basic limits. Thus, the concern is raised that the content described as appropriate for years K-2 does not challenge a significant portion of children in this age range.

Students in the Foundation year sort and classify objects and work with patterns based on observing natural patterns in the world around us, using materials, sounds, movements or drawings. They copy, continue and create patterns with objects and drawings (ACMNA005). In Years 1 and 2 students investigate simple number patterns in the numeration system through activities such as skip counting. They investigate and describe number patterns formed by skip counting and patterns with objects (ACMNA018).

The development of the concept of pattern might have been articulated with depth and continuity. The notion of pattern as a 'unit of repeat' could be made explicit in a sequence extending beyond simple repetition to include complex repetitions and the representation of the structure of patterns, albeit using invented symbolism. Pattern as 'unit of repeat' is interrelated with counting in multiples (or skip counting), repeated addition and early multiplicative thinking; this links with formal multiplication and proportional reasoning. Yet these fundamental concepts lack attention in the content descriptors. The opportunity to include the development of growing patterns, and functional thinking also seems to have been lost despite the fact that exploration of growing patterns supports the development of number patterns. Early algebraic thinking also includes the notion of equivalence but this does not appear until much later in Years 5 and 6. Important relationships such as developing commutativity and associativity are not made explicit which are important in modeling the structure of simple additive number sentences and word problems.

In comparison, the research-based National Council for the Teachers of Mathematics (NCTM) (2006) *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics* provides more explicit descriptions of, and makes direct connections between number and operations and algebra, and other 'focal points' in geometry, measurement and data analysis. Beginning Pre-Kindergarten student expectations are more advanced than for Patterns and Algebra in the *Australian Curriculum: Mathematics*. Number Operations and Algebra are often paired as a Focal Point and Connection. As well, the Algebra content is described separately providing a curriculum map that makes both the connection explicit as well as the specific place of algebra.

In essence, in Pre-K and K, students are expected to analyse how both repeating and growing patterns are generated; in Grade 1, they are required to "illustrate general principles and properties of operations, such as commutativity using specific numbers", and by Grade 2 "develop an understanding of invented and conventional symbolic notations, and describe qualitative and quantitative change" (NCTM, 2006, p. 24).

As early as Kindergarten, the Curriculum Focal Points and Connections describe patterns as "preparation for creating rules that describe relationships" and by Grade 1 "Children use mathematical reasoning…commutativity and associativity and beginning ideas of tens and ones to solve two-digit addition and subtraction" (NCTM, 2006, p. 25). In Grade 2, the interrelationships with number patterns are encouraged: "In Grade 2, through identifying, describing and applying number patterns and properties in developing basic facts, children learn about other properties of numbers and operations" and "develop, discuss and use efficient, accurate and generalisable methods to add and subtract multidigit numbers" (NCTM, 2006, p. 27)

Patterns and Algebra in Years 3–6

By Years 3 and 4 the *Australian Curriculum: Mathematics* content descriptors state that children are to be engaged in describing, continuing and creating number patterns that involve simple numerical operations with whole numbers and they describe patterns with numbers and identify missing elements (AMNA035). They describe, continue, and create number patterns resulting from performing addition or subtraction (ACMNA060) and solve problems by using number sentences for addition or subtraction (ACMNA036) and rational numbers (Years 5 and 6). The use of number sentences to represent word problems and finding unknown quantities is also introduced in Years 4 to 6. By Year 6 students "begin" algebraic thinking by describing the rules used to create number sequences. They continue and create sequences involving whole numbers, fractions and decimals and describe the rule used to create the sequence (ACMNA133). They explore the use of brackets to order operations and to write number sentences (ACMNA134).

Fundamentally, Years 3-6 are critically important for developing and consolidating the early understanding of structural relationships developed in the K-2 years. For example, the following two content descriptors from Number and Algebra in Years 4 and 5 could be used by teachers to extend the fundamental idea of equivalence, which can be introduced very early and is such an important concept for the later years.

> Year 4: Use equivalent number sentences involving addition and subtraction to find unknown quantities (ACMNA083)

> Year 5: Use equivalent number sentences involving multiplication and division to find unknown quantities (ACMNA121)

Consider number sentences such as $39 - 15 = 41 - \square$, or $5 \times 18 = 6 \times \square$. Teachers might use examples like these and interpret the content descriptors in one of two contrasting ways. First, the emphasis might be focused on using computation and equivalence to obtain a correct answer. In this situation teachers would encourage students to simplify each number sentence by calculating the value of the known pair of numbers, [24 in the subtraction sentence, and 90 in the case of the multiplication sentence]. They may then ask what unknown number on the right hand side will be needed to give these results, leading to 17 for the subtraction sentence and 15 for the multiplication sentence. It might be possible to some teachers to argue that this is all that is needed. However, this approach to teaching would neglect important opportunities to extend students' understanding of equivalence and its embodiment in different operations.

Second, an altogether richer approach would be to look more deeply at the structure of these and related equivalent number sentences; looking especially at how the direction of compensation changes according to the operations involved. In this kind of approach, students are encouraged to look at the numbers either side of the equivalent sign and to refrain from calculating. Some students may express this reasoning verbally, using rich and varied forms of mathematical thinking such as: "*Because 41 is two more than 39, I have to put a number that is two more than 15 in order to keep the same difference*". Other students may express their thinking by using arrows to connect related numbers, 39 to 41 and 15 to the unknown number, concluding that it has to be two more than 15 "to keep both sides the same". Similar forms of reasoning may be expressed in different forms of words; some students may explicitly use words such as "equivalent" or "to keep both sides the same". In all these cases, students are clear that they are dealing with equivalent differences. These students notice that the direction of compensation used the case of subtraction or difference operates in the opposite way to sentences involving addition. Likewise, for the multiplication sentence, students will be encouraged to notice that since 18 is three times the value of 6, the missing number has to be three times 5 in order to maintain equivalence. Students need to notice that because this sentence is about multiplication, the fact that the 6 is one more than 5 is not important, whereas the multiplicative relationship between 6 and 18 is the key to a solution.

These approaches focus on important and generalisable features of sentences involving the same operations. These features are intended to support students' computational fluency, and also to prepare them for algebraic thinking (Mason et al., 2009). These possibilities are quite new to many Australian primary and junior secondary teachers. We need to ask how well the new *Australian Curriculum: Mathematics* will open up teachers' vision to these ideas.

Another aspect that needs to be addressed is the inclusion of functional thinking. Research shows that young children are capable of thinking functionally. Functional thinking not only illuminates the relationship between the operations and their inverses but underpins much of algebraic thinking in the later years. Even young children have been found to understand situations involving co-variation. This a major gap in the *Australian Curriculum: Mathematics* and overlooking this important aspect in the content for primary students is perhaps the most serious omission in Number and Algebra.

By comparison, the NCTM (2006) *Curriculum Focal Points* Algebra are far more challenging. From Grade 3, "Children use number patterns to extend their knowledge of properties of numbers and operations for examples when they build foundations for understanding multiples and factors" (p. 28). For Grades 3–5 the focal points describes algebra as "understanding properties of multiplication and the relationship between multiplication and division as a part of algebra readiness". Through the creation and analysis of patterns and relationships involving multiplications and division…students build a foundation for later understanding of functional relationships by describing relationships in context with statements such as, "the number of legs is 4 times the number of chairs"(NCTM, 2006, p. 29). The expectations at Grades 3–5 describe algebra as requiring students to describe extend and make generalisations about geometry and numeric patterns and represent and analyse patterns and functions using words tables and graphs; identify properties such as commutativity, associativity and distributivity; represent

the idea of variable as an unknown quantity using a letter or symbol and express mathematical relationships using equations. By Grade 6 they are expected to write, interpret and use mathematical expressions and equations.

The development of mathematical concepts and proficiencies appropriate to Grade 6 requires an understanding of the structure of mathematical concepts, and the relationships between them (Mulligan & Mitchelmore, 2009), and acquisition of processes of abstraction, symbolisation and generalisation that are peculiar to mathematics (Mason, et al., 2009). The proposed content sequence for the *Australian Curriculum: Mathematics* does not do enough to highlight this fundamental idea of structure, in a way that exemplifies meaningful development. The notion that students do not 'begin' algebraic thinking until Year 6 explains to some extent why the content sequence from Foundation to Year 2 is limited. In comparison, the NCTM *Curriculum Focal Points* promotes the development of algebraic thinking from Kindergarten. It makes explicit the importance of the structural development of mathematical relationships, expressions and their connectedness to each other and other concepts as a 'big' unifying idea.

Another issue is that the content descriptors in Patterns and Algebra are developed sequentially, i.e. one after the other when they could be developed simultaneously. For example repetition is developed prior to introduction of equivalence and function. New research evidence demonstrates that several aspects of patterning and early algebraic thinking can be developed simultaneously with important links made between different concepts (Williams, 2010, Recommendation 24).

Algebra in the Secondary Years

It is generally accepted that all students should have access to algebraic reasoning (Yerushalmy & Chazan, 2008) since the ability to reason algebraically is a prerequisite for participation in higher levels of mathematics and it is important for access to many fields of employment that underpin our national economic growth (MacGregor, 2004).

Three decades ago Küchemann (1981) indicated that many students think of an algebraic symbol such as '*x*' as an unknown quantity and few consider the possibility that an unknown symbol can be a variable having multiple values. One reason for considering '*x*' as an unknown number instead of a variable may lie in students' previous arithmetic experience. Bednarz and Janvier (1996) found that difficulties in the transition from arithmetic to algebra stem from the differing nature of problems presented in each field and the various procedures used to solve these problems. In arithmetic calculations, students move from a known quantity to an unknown quantity. However, algebraic problems proceed from unknown to known quantities and are designed to indicate the relationships between the variables.

Students who interpret letters as specific unknowns or unknown quantities rather than as generalised numbers often learn the processes of evaluation and symbol manipulation without assigning any meaning to the letters involved (Booth, 1995). Misconceptions about variables are also linked to concepts like equivalence (Knuth, Stephens, McNeil, & Alibali, 2006; Perso, 1991). Learning and teaching in algebra therefore needs to focus more clearly on developing an understanding of variables instead of giving so much emphasis to symbol manipulation and equation solving (Warren, 2003).

Patterns and Algebra and Linear and Non-Linear Relationships in Years 7-10

In the *Australian Curriculum: Mathematics*, students in Year 7 continue the work begun in the primary years on generalising arithmetic properties and operations (ACMNA177) and the concept of a variable is established as a way of using letters to represent numbers in algebraic expressions and word descriptions (ACMNA175). Students create algebraic expressions and evaluate them by substituting values for each variable (ACMNA176).

The opportunity for students to develop a sound conceptual understanding of variables in Year 7 before they proceed to more abstract symbol manipulation is a positive step forward. However, it seems that students are expected to learn algebraic skills before using them in authentic situations and it may be preferable to adopt a problem-solving approach instead. For instance, students could take a worded problem and tease out the unknown quantities and the mathematical operations that link them to arrive at a solution without the need for formal algebra. Perhaps this could be more usefully done using a spreadsheet where the process of entering operations into the cells becomes a precursor to algebraic thinking. In this way, students will be better able to appreciate where the algebra they are leaning is applied.

In Year 8, students learn to expand (ACMNA190), simplify (ACMNA192) and factorise (ACMNA191) algebraic expressions based on simple operations and numerical factors. The notion that the laws applying to numbers can be generalised using variables is also reinforced. The laws for variables involving positive integral indices and the zero index are introduced in Year 9 (ACMNA212) and there are some good links made from the algebraic indices to number work in Indices and Scientific Notation in the real numbers sub-strand and to simple interest calculations in the money and financial mathematics sub-strand.

Students also work with binomial products and learn to collect like terms in algebraic expressions (ACMNA213). Factorisation is extended in Year 10 through

work with algebraic factors (ACMNA230) and index laws are used to simplify algebraic products and quotients (ACMNA231), leading to the need for negative indices. Students in Year 10 also study simple algebraic fractions (ACMNA232), extend their work with binomial products to investigate special binomial products based on perfect squares and the difference of squares, and they learn the method of completing the square to solve monic quadratic equations (ACMNA233). The content associated with substitution into algebraic expressions is also extended to substitution into formulas to determine the value of an unknown quantity (ACMNA234). The algebra for Year 10A covers polynomials and includes algebraic long division and the factor and remainder theorems (ACMNA266).

The Linear and Non-Linear Relationships sub-strand begins in Year 7 with an introduction to the Cartesian plane which focuses on plotting points with given coordinates and on finding the coordinates of a given point (ACMNA178). Simple linear equations are solved using concrete materials, the balance method, and strategies such as backtracking, guess and check and improve (ACMNA179). Students also investigate travel graphs and interpret other straight-line graphs based on real-life data.

In Year 8, students plot linear relationships with and without the use of technology (ACMNA193), learn to solve linear equations using algebraic and graphical techniques, and continue their work solving linear equations using a variety of methods (ACMNA194). Distance (ACMNA214), midpoint and gradient (ACMNA294) are introduced in Year 9 using a range of approaches, including graphing software. Students sketch linear (ACMNA215) and simple non-linear relations such as parabolas, hyperbolas and circles (ACMNA296), with and without the use of technology. They also determine the equations of lines from tables of values and graphs.

In Year 10, students solve problems based on linear equations (ACMNA235) and learn about linear equations involving simple algebraic fractions (ACMNA240). They solve quadratic equations and identify the link between the x-intercepts of the parabola and the roots of the corresponding quadratic equation (ACMNA241). Equations are used to represent word problems and students learn how to solve linear inequalities, graphing the solutions on a number line (ACMNA236). Linear simultaneous equations are introduced using algebraic and graphical means to solve them (ACMNA237). Co-ordinate geometry is extended through an investigation of the properties of parallel and perpendicular lines (ACMNA238) and an exploration of the connection between algebraic and graphical representations of simple quadratics, circles and exponentials (ACMNA239).

The algebra content for Year 10A focuses on work with non-linear graphs such as parabolas, hyperbolas, circles and exponentials to study transformations of these graphs by connecting their graphical and algebraic representations (ACMNA267). The work on equations is extended to simple exponential equations based on population growth (ACMNA270), as is the polynomials topic through sketching a range of curves (ACMNA268), investigating the features of these curves using technology, and solving monic and non-monic quadratic equations by factorising (ACMNA269). However, it is questionable whether the extra algebra content covered in 10A will be sufficient preparation for those students who intend to study mathematics at the highest level in Years 11 and 12 since a solid foundation in algebra is essential when learning advanced mathematical topics.

Research on Technology use in Patterns and Algebra and Linear and Non- Linear Relationships

More than a decade ago, Clements (1999) highlighted the benefits of virtual manipulatives for classroom use. For example, virtual Pattern Blocks have colours that can be changed and they can be 'snapped' into position unlike concrete material and they "stay where they're put" (Clements, 1999, p. 51). The role and benefits of software and virtual tools, such as *Kidpix* and *Kidspiration* (Hong & Trepanier-Street, 2004) and virtual manipulatives (Highfield & Mulligan, 2007; Moyer, Niezgoda & Stanley, 2005) show that each of these tools has potential advantages for developing patterning skills. Yet there is scant attention to the integration of these readily available resources to support the development of content and acquisition of curriculum proficiencies such as understanding and reasoning.

When using virtual manipulatives and dynamic interactive software it appears that making judicious and wise choices in software selection is key to mathematical success. For example while there is a large range of virtual manipulatives, only some, such as the 'Pattern Block' manipulatives are open-ended and have considerable mathematical potential. Other virtual manipulatives, such as 'Complete the Pattern' are not as powerful mathematically as they offer a closed questioning style and do not allow transformation or manipulation that is necessary for early mathematical development.

In the secondary school, technology can significantly change the nature of the opportunities for algebraic activities of conceptualising, generalising, and modeling (Thomas, Monaghan, & Pierce, 2004). There has been considerable research on the role of technology in enhancing students' development of the function concept and their ability to confidently apply processes of symbol manipulation (Kieran & Yerushalmy, 2004).

Spreadsheets provide an intermediate step between arithmetic and algebra, especially in assisting students to develop their understanding of a variable as

something that varies (Haspekian, 2005; Yerushalmy & Chazan, 2008), and they allow students to begin their study of functions using tabular representations (Confrey & Maloney, 2008). The dynamic and interactive nature of graphing software can support students' control of the appearance of a graph (Ainley, 2000) so that they can explore many features of graphs and are more likely to investigate hypotheses to discover new relationships (Ruthven, Deaney, & Hennessy, 2009). Graphing software also affords both local and global views of graphs, and it allows students to explore multiple representations to highlight different graphical features (Heid & Blume, 2008). Computer Algebra Systems (CAS) permit a richer treatment of functions, both as processes and as mathematical objects in their own right. Students can use CAS to make connections between graphic, numeric and symbolic representations of functions, and reason about them (Zbiek & Heid, 2008).

Access to CAS tools that allow automatic generation and operation on symbolic expressions highlights the need for a reconceptualisation of the role of symbol manipulation (Yerushalmy, 2000; Noss, 2001). For instance, there could be greater emphasis placed on word problems and practical applications rather than solving symbolic equations (Nunes-Harwitt, 2004/05). CAS and other symbol manipulation programs will necessitate new emphases, such as the need to deal with equivalence of symbolic expressions (Artigue, 2002). At the same time, new difficulties for students may arise, such as the need to distinguish variables from parameters (Drijvers, 2003).

Technology in Patterns and Algebra and Linear and Non-Linear Relationships Sub-strands *The Australian Curriculum: Mathematics* advocates that mathematics classrooms will make use of all available ICT in teaching and learning situations with the expectation that a developmentally appropriate range of technological resources might be aligned with the content descriptors and achievement standards (Goos, this volume).

Although there is some reference to the use of technologies for learning in the Number strand it is omitted from the Patterns and Algebra sub-strand from Foundation to Year 7 and is presented as optional rather than a mandatory requirement in Years 7 to 10. The integration of technologies could have enhanced the content by providing links to a rich range of technological tools and resources explicitly supporting the implementation of Patterns and Algebra. Exemplars of web-based resources, digital learning objects and other software from the pre-Foundation years might have highlighted the affordances and limitations of these technologies for mathematics learning.

In the Patterns and Algebra sub-strand, patterning forms a 'thread' where children engage in simple through to complex patterning, primarily developed as repetition. Generally these patterning experiences imply the use of concrete materials and representations of patterns drawn or made by the children using traditional media. Without explicit reference to the use of digital resources in developing patterning and early algebra concepts this curriculum does little to encourage teachers and students to maximise opportunities to create or represent patterns in an on-screen environment.

Using Technology in Years 7-10

Guiding students' development of algebraic reasoning in a technology-rich environment will therefore require a curriculum that gives more prominence to the structure and key features of algebraic expressions. Students will need to correctly enter expressions into a CAS, efficiently scan the working and results to identify any possible errors, and interpret the output. Artigue (2002) has developed the notion of *technique* to describe a more expanded view of symbolic manipulation required in a CAS environment. Similarly, Pierce and Stacey (2001) highlight the importance of *algebraic insight* or the mathematical thinking students need to do algebra with CAS, including algebraic expectation and the ability to link representations. Similarly, whilst technological tools support generalisations, they also affect the nature of those generalisation activities (Hershkowitz, Dreyfus, Ben-Zvi, Friedlander, Hadas, Resnick, Schwarz, & Tabach, 2002). For instance, the rapid generation of a large number of examples using technology can afford students greater opportunities to identify patterns and generalise their results, such as when observing how the changes in values of parameters of functions can affect their graphs.

The availability of technological tools can serve as a powerful catalyst for rethinking the traditional algebra curriculum in the secondary years (Stacey, Asp, & McCrae, 2000). For example, CAS facilitates a more generalised approach to the teaching of equations (Ball, 2001) and it can support the use of different types and combinations of functions that permit modelling of real life situations (Thomas, 2001). In these activities, symbolic reasoning assumes greater importance than manipulation and simplification of algebraic expressions (Heid, 2002; Heid & Edwards, 2001) since CAS coupled with dynamic construction tools allows students to link symbolic results to numerical and graphical representations rather than focus so heavily on abstract symbol manipulation.

Technology may provide more access to algebraic thinking and techniques for a greater number and variety of students, but can also present barriers for other students (Thomas, Monaghan, & Pierce, 2004). Hence, there is a need for algebra curriculum changes in the order of topics, their relative importance, and the way they are presented (Yerushalmy, 2005). In particular, more numerical solution methods for certain types of problems and realistic problem solving (as opposed to learning symbol manipulation techniques before trying to apply them) should be

emphasised. The introduction of technology facilitates investigation of multiple representations and also has the potential to change the way some aspects of algebra and functions are introduced. But care needs to be exercised since, for example, a sequential treatment of symbolic and graphical representations does not assist students in establishing links between them. Finally, there remains the question of how much algebra should continue to be done by hand and how much can be realistically achieved using technology (Flynn, Berenson, & Stacey, 2002).

The role of digital technologies and graphing software in the learning of algebra in Years 7 to 10A is somewhat problematic. Digital technology should be used when it improves teaching and learning efficiency. A static display, even one that results from student inputs, rarely adds meaning for the student whereas a dynamic display that is manipulated by student input is more likely to be central to the learning of new concepts. There are now emulators of graphic display calculators that can be installed on a computer to offer the added benefit of immediate feedback. Students and teachers can connect their calculator display to a digital projector so all members of the class can view it for the purposes of discussion and analysis.

Technology is not mentioned at all in the patterns and algebra sub-strand when it might have been advantageous to include technologies in order to strengthen students' ability to link numerical and symbolic representations of algebraic expressions. Technology is highlighted in many of the outcomes for Years 8 to 10A of the linear and non-linear relationships sub-strand through activities such as plotting graphs, solving equations by graphical methods, and connecting graphical and algebraic representations of functions. However, the approach adopted in recommending the use of technology is rather cautious and the use of technology could have been made more explicit. For instance, phrases such as "using a range of strategies, including graphing software" or "with and without the use of digital technologies" may suggest to teachers that the use of technology is supplementary to their needs rather than a central focus of classroom learning and teaching.

The explicit use of technology is not always articulated in the content descriptors. For example, Williams (2010) raises the question of why technology is not exemplified in the exploration of the relationship between quadratic functions and their graphical representations in Year 10. Given that students now have ready access to spreadsheets, graphic display calculators, and other computer software, it is questionable whether to wait until Year 10 before linking problem solving and algebra in a meaningful way. It appears that a very traditional approach to learning algebra underlies the *Australian Curriculum: Mathematics* and that digital technology is seen as something of an accessory.

Interrelationships Between Number and Algebra and the Proficiencies Strand

The proficiency strands of Understanding, Fluency, Problem Solving and Reasoning describe the actions in which students can engage when learning about the syllabus content. In the early sections of the chapter we have discussed various aspects of mathematical understanding connected to structural relationships between arithmetic and algebraic reasoning. The Problem Solving and Reasoning proficiencies are also of particular relevance to algebra, as is Fluency is terms of mental calculation and efficient computational strategies.

A focus on equivalence and patterning in the primary school years should have an immediate pay-off for fluency in calculation, including mental computation. For example, adding or subtracting 9, 99, 999 (as well as other numbers close to these, and using 'near decade' numbers such as 39, 49) is made easier and more fluent by knowing how sentences involving these numbers can be converted, for example, into equivalent or near equivalent sentences involving 100 and so on. A similar case can be made for multiplication and division.

Problem solving is the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively (ACARA, 2011). Students apply their mathematical understanding by planning how to solve unfamiliar problems and checking the reasonableness of their answers. In the primary years, problem solving is described as the formulation of problems based on authentic situations, the use of materials to model problems, and using number sentences to represent the problem situations. In the secondary years, problem solving is aligned more closely to specific content such as using algebraic and graphical techniques to solve simultaneous equations and inequalities in Year 10.

The direct link between Number and Algebra and the Reasoning proficiency strand should enable students to develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising (ACARA, 2011). Reasoning activities that relate to the algebra content thus involve creating and explaining patterns, and generalising number properties. Students are expected to transfer their mathematical understanding among different situations and develop notions of mathematical proof.

However, the development of informal proof is given superficial attention and could be developed as a 'bottom-up' strategy from the early years. It would be difficult for a teacher to trace the introduction and development of 'proof' in the Patterns and Algebra sub-strand year by year. The processes of justification and argumentation are integral to the notion of mathematical proof from the preschool years (Perry & Dockett, 2008), yet they are not described or linked to the content descriptors. The language of reasoning and proof may present particular challenges for students in the primary years. That is why the distinction between when it is appropriate to use the term 'prove' and terms like 'justify', 'explain', 'support' 'demonstrate' or 'convince' needs to be provided. If the curriculum is serious about developing students' skills in justification, argumentation and proof in preparation for more formal mathematics then these proficiencies need to be linked explicitly to Patterns and Algebra and assessed both informally and in assessments such as the National Assessment Plan Literacy and Numeracy (NAPLAN).

Challenges Facing Teachers and the Impact of Patterns and Algebra on the Development of Teachers' Pedagogical Content Knowledge (TPCK)

Some may even view the introduction of the pairing of algebra with number strands as difficult to align and integrate. The pairing of these content strands seems reasonable given that some categorisation of content will be necessary. But for primary teachers who have traditionally taught the number strand and its sub-strands focused on counting, place value computations, and fractions and proportional reasoning, the pairing with algebra may be difficult to conceptualise. What might be even more difficult is to trace the development of arithmetic structural relationships in terms of early algebraic reasoning from the early grades.

An experienced upper primary teacher questioned recently whether the Number strand had been given less emphasis to make way for algebra on the assumption that the Number content had been 'halved' to incorporate algebra. Moreover for teachers in the early grades, it is challenging to conceptualise what is meant by teaching early algebra. Traditionally this has been restricted to the development of patterning skills.

In particular the new curriculum presents challenges for teachers in the early and primary grades. The notion of unit of repeat, growing patterns, functions and equivalence are but some of the big conceptual ideas that required robust understanding in order to implement effective pedagogical strategies. In turn this will have implications for the transition from arithmetic to algebra in the secondary school.

Implications for Curriculum Implementation and Further

Within the last decade or so, a more coherent research base has evolved to inform the re-development of a scope and sequence from preschool through to Year 12 to incorporate early algebraic reasoning, and algebra within mathematics curriculum (Blanton & Kaput, 2003; Clements & Sarama, 2009; Jacobs et al., 2007; Mulligan et al., 2011; Papic et al., 2011; Warren & Cooper, 2008).

A number of new classroom-based studies are looking at ways that teachers can promote the development of early algebraic reasoning, structural relationships and generalisation in children's early mathematics learning. Research on effective professional development for pre-service and practicing teachers must be integral to this work. Two questions that the research raises are how teachers can develop a deeper understanding of why pattern and structure is critical to early algebraic thinking, and how they may overcome traditional perceptions that algebra is the exclusive domain of secondary school mathematics. "Some may react to the idea of introducing algebra in the elementary school with puzzlement and skepticism" (Carraher & Schliemann, 2007, p. 670). Another confounding problem that needs urgent research may be lack of teacher pedagogical content knowledge particularly for the early and primary professionals (Ball et al., 2008). More research is needed to identify underlying weaknesses in teacher pedagogical content knowledge and the development of relational thinking necessary for the teaching of algebra (Asquith et al., 2007).

In our discussion we have provided an outline of the *Australian Curriculum: Mathematics* Number and Algebra content strand, which does to some extent present a connected view of mathematics, by linking number with pattern for algebraic thinking. On the surface this provides a positive sense of expectation about the Number and Algebra strand, with the opportunity for greater conceptual and connected knowledge and the development of teaching practices that focus on relational thinking. The definitions of the Proficiency strands (understanding, fluency, problem solving and reasoning) indicate a curriculum that is concerned about mathematics as patterns, relationships and generalisations, and not just facts, skills and rules. However, the structure and depth of the content, as presented through the descriptors appear to lack the coherence and connectedness that the document promises, which results in some concerns about the implementation and effectiveness of the strand overall.

In grappling with the implementation of the *Australian Curriculum: Mathematics* further fundamental questions are raised: What is the appropriate content and sequence for teaching algebra in the K-6 (primary) school that students are capable of achieving, and what is its relative importance in the Number and Algebra strand across all grades?. What relation does it have to the other two strands–Measurement and Geometry and Statistics and Probability? Moreover is early algebra really about developing proficiencies in reasoning, abstracting and generalising? What role does the early introduction of algebra play in the mathematics curriculum overall? What forms of assessment of early algebra, beyond patterning skills, will students in the primary years encounter i.e. NAPLAN, standardised assessments? How will the new Number and Algebra scope and sequence influence the way teachers approach

traditional pedagogy focused on number skills and operations? How will teachers come to change their practice to incorporate the wide range of technological tools available for the contemporary teaching of number and algebra in the 21st century?

What we may need in the *Australian Curriculum: Mathematics* is not merely adding early algebra to the number content strand for the primary school years, or beginning algebra earlier in the mid or upper primary grades. Early algebraic reasoning is not formal algebra introduced earlier. What is perhaps needed is a reconceptualisation of the development of algebraic thinking and the interrelationships between algebra and arithmetic and other aspects of mathematics. Only then can curriculum developers weave it appropriately throughout the mathematics curriculum.

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Chapter 4
Perspectives on Geometry and Measurement in the Australian Curriculum: Mathematics

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The Australian Curriculum: Mathematics presents the measurement and geometry content strands together in order to emphasise their relationship to each other and highlight their practical relevance. In this chapter, we examine the connectivity between the measurement and geometry sub-strands and the interconnectivity with other content strands in the document. We analyse the terminology and language used throughout this content strand; evaluate the framing and structure of the strand, noting the lack of reference to visual and spatial reasoning; and question whether current assessment practices are congruent with the measurement and geometry strand. One of the most positive aspects of this strand was the potential for teachers to develop rich, conceptually-connected learning opportunities. A major area of concern was the lack of reference to visual and spatial reasoning within the content strand. Seen as a critical and integrated aspect of both sub-strands, the lack of attention afforded to such reasoning processes may impact on the way teachers enact the Curriculum. We suggest a significant and sustained professional development program to accompany the implementation of the Curriculum to ensure the connectivity between content strands is made explicit.

Introduction

In this chapter we describe the framework of the measurement and geometry content strand of the new Australian Curriculum for mathematics and its impact on curriculum implementation in the classroom. We pose a number of questions within this chapter, specifically seeking to determine what kinds of knowledge and skills in measurement and geometry are valued in the document. Furthermore, we explore the mathematical foundations of the Curriculum by highlighting the emphasised concepts and understandings that are privileged in the document and the extent to which the decisions to include these concepts and understandings have strong

In B. Atweh, M. Goos, R. Jorgensen & D. Siemon, (Eds.). (2012). Engaging the Australian National Curriculum: Mathematics – Perspectives from the Field. Online Publication: Mathematics Education Research Group of Australasia pp. 71-88.

research and theoretical foundations (Reid, 2005). Moreover, we examine the extent to which this Curriculum supports students and teachers in a rapidly changing society and provide suggestions for further curriculum development in this content strand.

As levels of accountability for both teachers and school systems increase, aspects of the curriculum that are not easily testable (and thus measurable), such as open-ended problem solving and practical applications of mathematics, are at risk of being squeezed out of the classroom curriculum. As Dimarco (2009) asserted, teachers struggle to facilitate open-ended problem-solving tasks when the focus on national testing is overly emphasised. In relation to measurement and geometry, it is much easier to test basic skills and understandings (under pencil-and-paper conditions) than it is to assess students' mathematical cognitive problem-solving processes (English & Sriraman, 2010). As a consequence, future curriculum standards could be altered and lowered to allow more students to perform well in standardized testing (Hattie, 2005). In parallel with this shift, the number content strand of the curriculum has been awarded increased attention, both from a teaching and learning and an assessment perspective (Verschaffel, Greer, & De Corte, 2007). Hence, it is imperative that the more practical strands of mathematics, namely measurement and geometry, are afforded the attention they deserve. As Owens and Outhred (2006) explained, the measurement and geometry content strand provides rich opportunities for visual reasoning, representations of mental schemas and engagement with physical objects and representations. Thus, practical nature of this strand promotes advanced mathematics reasoning.

An additional challenge for curriculum design and implementation is the concern raised about teachers' Pedagogical Content Knowledge (PCK) (Hill, Rowan & Ball, 2005) and the extent to which teachers rely on support documents and textbooks to present mathematics content. Generally, curriculum documents, and certainly the Australian Curriculum for Mathematics, presume that teachers are able to make connections between and across mathematics strands in order to promote mathematics thinking. Recent literature measuring teachers' pedagogical content knowledge identified a deficiency in their ability to progress students' knowledge to develop further mathematical understanding after assessment (Callingham, 2010; Watson, Callingham, & Donne, 2008; Watson, Callingham, & Nathan, 2009). Furthermore, a broad literature base indicates that teachers' content knowledge is limited (da Ponte & Chapman, 2008; Vinson, 2001; Weiss, 1995) and that many teachers have difficulty relating and separating concepts of length, area and volume within the measurement and geometry content strand. For example, many teachers incorrectly assume that when the perimeter of a figure increases, so too does the area of the figure (Ma, 1999); and that as the length of the sides of a square double, so does its area and volume (Tierney, Boyd, & Davis, 1990). Such limitations in content knowledge influence the effectiveness of PCK and therefore disrupt links between curricula and teaching and learning. Consequently, any evaluation of the Australian Curriculum needs to be undertaken within the context that teacher's content knowledge and pedagogical content knowledge is generally limited.

This chapter is structured under three headings: 1) a description and synthesis of the content; 2) the representation structure and framing of the strand; and 3) the assessment practices: backward mapping from national assessment. Each of these sections provides commentary of aspects of the Curriculum and integrates research findings into the argument. A discussion and conclusions are then presented.

A Description and Synthesis of the Content

Connectivity Within and Across the Strand

Within any mathematics curriculum, there has been a concerted push to ensure that measurement and geometry understandings are introduced, developed and applied in connected ways. In both primary (Bobis, Mulligan, & Lowrie, 2009; Zevenbergen, Dole, & Wright, 2004) and secondary (Goos, Stillman, & Vale, 2007) contexts researchers have advocated for concepts and understandings to be simultaneously presented to students in order to foster deeper levels of reasoning. As Battista (2007, p. 891) acknowledged "understanding measurement [and geometry] requires an integration of procedural and conceptual knowledge". Without a connected curriculum there are few opportunities for students to confront the relationships between and among concepts.

Recent mathematics education research has identified the need for teachers to emphasise connections between subject matter that have the same conceptual underpinnings (Bobis, Mulligan, & Lowrie, 2009) but also to establish sound understandings across topics. As Reid (2005) explained:

> no matter how knowledge-content is organised in the official curriculum, the decision about whether or not to work within or across discipline boundaries is a professional one that is taken at the classroom level as teachers work through the issue of how best to develop the capabilities. (p. 63)

With respect to measurement and geometry concepts, Booker and Windsor (2010), for example, maintained that students should represent and solve related problems in a variety of ways in order to articulate and generalise their solutions. They argued that aspects of measurement and geometry understandings constructed in primary school had the facility to help students "to construct algebraic notation in a meaningful way through their representations using materials, diagrams, models, tables and graphs in their search for patterns and

generalizations" (p. 418). Other studies have demonstrated the connectivity between measurement and geometric understandings and other strands including the number concepts (Bragg & Outhred, 2004) and algebraic reasoning (Clements & Battista 1992). Teachers require more explicit direction on how to make links across content strands in the Mathematics Curriculum to ensure that existing topics of early mathematics are tightly interwoven and foundations are developed for subsequent learning (Carraher, Schlieman & Schwartz, 2008).

So we pose the question, to what extent does the Measurement and Geometry strand of the curriculum highlight the relationships not only within the strand, but also across the other strands? In other words, are concepts and understandings addressed in isolated or connected ways? The following section provides a synthesised year level description of how closely aligned sub-strands are presented throughout this content strand.

Initially, we look at the connection between measurement and geometry concepts in the curriculum. Figure 1 highlights the connection (or not) between the sub-strands. If we consider the overarching framework of this strand, the dotted lines represent connections across sub-strands within each Year level. In Foundation and Year 1, there are no explicit connections between the sub-strands. Perhaps this is based on the assumption that students have acquired limited prior knowledge of such concepts at an early age—however, recent literature demonstrates young children's ability to use measurement vocabulary and apply it to pertinent situations in holistic and sometimes relatively sophisticated ways (MacDonald, 2010; Sarama & Clements, 2004). More problematic is the lack of explicit connectivity between *shape* and *location and transformation* until Year 5. At Year 5, there seems to be many pedagogical opportunities for connected and rich learning situations. For example, in Year 5, the connections between *shape* and *location and transformation* can be seen from the outcome: "Describe translations, reflections & rotations of 2D shapes**"** and the connection between *shape* and *geometric reasoning* is identified through the outcomes: "Apply the enlargement transformation to familiar 2D shapes & explore the properties of the resulting image compared with the original"; and "Estimate, measure & compare angles". Yet a worrying dimension to this analysis is the fact that many of the connections seemingly established in Year 5 appear not to be reinforced in Year 6. In fact, the connections across sub-strands are limited in Year 6 (and the only other instances of this lack of connectivity occur in the first two years of schooling) (see Figure 1). We trust that Year 6 has not become a revision year. There have been ongoing calls for curriculum designers and classroom practitioners to provide rich tasks for students to engage with (van den Heuvel-Panhuizen, 2010) in order to provide depth and scope within concept development. Measurement and geometry understandings are highly suited to such opportunities and rich-task development (Bobis, Mulligan, & Lowrie, 2009).

Figure 1 also displays the scope and sequence of sub-strand content across years. Three of the five sub-strands are introduced during the Foundation year, with Geometric Reasoning introduced in Year 3 and Pythagoras and Trigonometry content in Year 9. The *using units of measurement* sub-strand is common from Foundation through to Year 10 and this has links with *shape* (especially in primary school) and then *geometric reasoning* (in secondary school). Interestingly, *shape* and *location and transformation* cease to be sub-strands in Year 7 and the connectivity is then between *geometric reasoning*, *units of measurement* and, in years 9 and 10, *Pythagoras and trigonometry*. The spiraling effect of the curriculum allows students to build on concepts based on previous knowledge. Although we hoped for far reaching connections across sub-strands, it could be argued that there are relatively sound connections within and between sub-strands in the Measurement and Geometry content strand. Nevertheless, some explanation as to why sub-strands disappear at certain stages or levels must be articulated. Otherwise, the establishment of mathematics understandings within these sub-strands (e.g., links between *shape* and *geometric reasoning*) will be lost. Figure 1 presents the scope and sequence of each of the sub-strands of the Measurement and Geometry strand and highlights the connectivity between them. Our representational interpretation of the curriculum illustrates (through dark bolded lines) where strands begin and end. The dotted lines highlight contents specific links across strands with information about when these links occur by year.

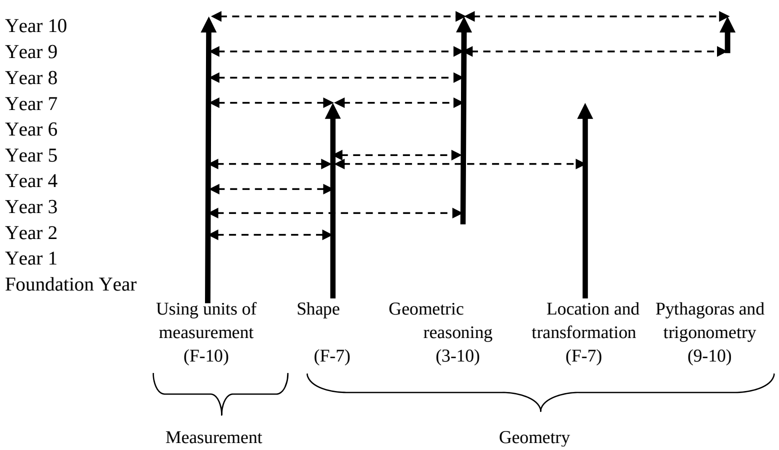

Figure 1. Connections between the Measurement and Geometry content strand throughout the Australian Curriculum

It is important to note that at a meta-level, connections across content strands have been formed. For example, although *location and transformation* ceases in Year 7, there is a direct developmental link within this understanding and that of *linear and non-linear relationships* (which are described in the Number and Algebra strand). There are other links within and across content strands which also drive a connectivity agenda, however, we are concerned that the frameworks currently established do not provide teachers with easy access to such models, nor a holistic view of the curriculum.

It is certainly the case that the approach of the Australian Curriculum is to build on students' understandings within topics and content areas and have them relate to each other once understandings have been separately scaffolded and understood. By way of example, the definition of a square is as follows*: A square is a quadrilateral that is both a rectangle and a rhombus* (ACARA, 2011, p. 71). This definition highlights the hierarchical properties of the quadrilateral family rather than isolating a shape as a discrete prototype (Bobis, Mulligan, & Lowrie, 2009)—and indeed emphasises the importance of establishing connected understandings within a single sub-strand. Most definitions of a rhombus describe its properties as *a four-sided shape with four equal sides with opposite angles equal* (O'Brien & Purcell, 2004). Never do these definitions have a shape of a square as an example of a rhombus—the representations are always of a figure with two obtuse and two acute angles. Other examples would include teaching perimeter and area, and area and volume understandings through an integrated approach. However, as the document stands we do not have access to the level of depth required to ascertain whether such learning experiences are to be promoted in future supplementary documents (e.g., units of work). Nevertheless, what we do know is that scant attention is given to such integrated teaching in current state curricula.

Although there is a strong indication that such connected learning opportunities are endorsed in the Curriculum, substantial and sustained professional learning opportunities need to be provided for teachers (Bezzina, Starratt, Burford, 2009; Reid, 2005; Reid, 2010). There is a view that the Australian Curriculum may be an enabling process that changes practices in the classroom (Tonkin & Wilkinson, 2010) however, the nexus between policy and practice needs to be strong (Green, 2010). If this fails to happen, the curriculum will become fragmented with concepts being introduced and reinforced in isolated ways. This "warning" is particularly pertinent to the Measurement and Geometry content strand since most learning opportunities, in both primary and secondary school contexts, will revolve around the Number and Algebra content strand. This practice will occur irrespective of curriculum initiatives—and possibly even if professional learning opportunities at pre-service and in-service levels are saturated with experiences which enhance the

Measurement and Geometry strand—given the overwhelming focus of Number and Algebra in national assessment and policy frameworks (Lowrie & Diezmann, 2009).

To this point, the Australian Curriculum needs to have better scaffolds for classroom teachers to engage students within and across content strands. We argue that this could be achieved through the four proficiency strands and especially through the *understanding* and *reasoning* proficiency areas. The description of the proficiencies identify that students "make connections between related concepts and progressively apply the familiar to develop new ideas" (*understanding* proficiency) and "when they adapt the known to the unknown, when they transfer learning from one context to another" (*reasoning* proficiency) (ACARA, 2011, p. 3). We argue that such statements may not be enough[1]. Anderson (2010) asserts that the mathematics draft curriculum has failed to clearly link the connection between the mathematical content prescribed; and the *proficiencies* and actions associated to working mathematically within content areas, particularly in reasoning and problem solving. It is Anderson's (2010) view that teachers will need support in teaching reasoning and problem-solving skills if the outcomes of the Australian Curriculum are to be fulfilled. From our perspective, ACARA should provide meta-models which not only describe the connectivity between sub-strands within the Measurement and Geometry content strand, but also models which highlight the connectivity of sub-strands across the three content strands.

Readiness

To date, much debate concerning the Australian Curriculum for Mathematics has centred upon the inclusion of the content knowledge and proficiencies students should possess and be equipped with to flourish in an increasingly global society (Bezzina, Starratt & Burford, 2009; Reid, 2010). An area that needs further exploration revolves around students' readiness to engage with the content knowledge prescribed in the curriculum. Recommendations from the National Numeracy Review (Council of Australian Governments [CoAG], 2008) suggest:

> That from the earliest years, greater emphasis be given to providing students with frequent exposure to higher-level mathematical problems rather than routine procedural tasks, in contexts of relevance to them, with increased opportunities for students to discuss alternative solutions and explain their thinking. (p. xii)

We argue that students bring with them significant measurement and geometry knowledge when they arrive at school. This knowledge often exceeds knowledge

[1] We acknowledge that subsequent documentation and support documents may well be developed by State jurisdictions.

beginning students have on other mathematics strands and thus there is more potential to engage in deeper levels of understanding at an earlier age in this strand (Highfield, Mulligan & Hedberg, 2008; Owens, 2002).

MacDonald (2010) considered the impact of young children's informal measurement experiences on their ability to compare like attributes; directly compare objects; use appropriate language to describe measurement attributes in comparison; and order objects using direct comparison. She discovered Kindergarten students without any formal learning in measurement were capable of demonstrating comparison between similar and different objects, and at the most sophisticated level, comparison between more than two objects. As Barrett and Clements (2003, p. 515) argued, "measurement takes its meaning from comparisons of real objects; as such, children's schemes for measuring linear objects become more sophisticated when they are grounded in realistic situations based in comparison". MacDonald (2010) claimed students' positive outcomes in comparative measurement generate confidence, and consequently, positive self-efficacy when encountering measurement curriculum material in classroom settings. Although the notion of confidence building is critical at every grade level, we emphasise it here (in the early years of schooling) since curriculum content appropriateness begins with what prior understandings and skills students have acquired—and these foundations build any curriculum.

Several researchers (including MacDonald, 2010; Bobis, Mulligan & Lowrie, 2009) have shown that students naturally compare the measurement of objects to their own body. Consequently, the measurement outcomes of the Australian Curriculum for Mathematics should present opportunities for students to use their resources and prior experiences to demonstrate their knowledge of informal comparison. This is beneficial to students' understanding as Bush (2009), Jacobbe (2008) and O'Keefe and Bobis (2008) agree that students experience difficulty when measuring an attribute in regards to choosing appropriate units to measure different attributes; and the comparison of attributes. Students' understanding of attributes will develop over time provided teachers use a variety of measurement tools for students during tasks involving the measurement of an attribute (Castle & Needham, 2007). At present we find that tools and concrete materials are not afforded the attention required in order for such development to take place. Perhaps these working mathematically characteristics are embedded within the four proficiencies but we would like shifts between informal and formal measurement to be horizontally rich rather than just vertically linked. In other words, students should be encouraged to discover and explore measurement and geometry understandings throughout their schooling rather than such experiences dissipating as students get older. The initial sub-strands of *using units of measurement*, *shape*, and *location and arrangement* (see

Figure 1) provide scope for rich learning opportunities, and we trust that support documents that accompany this Australian Curriculum will highlight the importance of embedding the four proficiencies into all sub-strands of this strand. Otherwise, progression through the curriculum will result in less engagement with realistic and practical measurement connections and move toward rule-based procedures that only have application for some students (see Ness & Farenga, 2007 for a description of how young students construct a sense of ratio through practical experiences from an early age).

Terminology and Language

In terms of mathematics terminology, particular words have been used to describe and represent mathematics concepts, symbols and images—and these words have been used interchangeably (e.g., flip and reflection) and differently (column graph and bar chart) within and between states syllabi. As we move toward an Australian Curriculum, such idiosyncratic and inconsistent usage is heightened. Although there has always been a transition between using appropriate mathematics terminology and using "everyday" word usage to describe specific mathematics concepts, it appears that such practices are not only unwarranted but also problematic. Communication is an essential ingredient when exploring mathematical ideas and relationships. As Muir (2006) argued, the use of estimation prior to measuring attributes allows students to practise communicating their expectations and theories in meaningful ways. Communication during measurement is viewed as a valuable tool to build students' use of mathematical language (Thom, 2002) and to consolidate conceptual understandings. Indeed The National Numeracy Review (CoAG, 2008) recommended:

> That the language and literacies of mathematics be explicitly taught by all teachers of mathematics in recognition that language can provide a formidable barrier to both the understanding of mathematics concepts and to providing students access to assessment items aimed at eliciting mathematical understandings. (p. xiii)

Hence, teachers must faciliate students' learning that everyday vocabulary, such as 'pie' or 'column', can adopt different meanings when used in the context of measurement and geometry in the classroom (Ernst-Slavit & Slavit, 2007; Lowrie & Diezmann, 2009). Moreover, it is much better to encourage the use of precise terminology with children from an early age to avoid confusion and inconsistencies as they developmentally progress through the curriculum. An example of this is the apparent interchangeable use of "features" and "properties" when describing geometric terms. Dawe and Mulligan (1997) found that over 10% of children failed to correctly respond to a Basic Skills Test mathematics item as they were unable to comprehend the wording of the task as opposed to a lack of mathematical

knowledge. Lowrie and Diezmann (2009) described the confusion that surrounds the use of the term "flip" when students are required to reflect a shape. Typically, students adopt a practical "everyday" definition to such terminology which can completely disrupt mathematics sense making. We note that terminology such as "slides" and "flips" are used in Year 2 and Year 3 and yet "translations" and the "reflections on" are terms used by Year 5. The same case could be made for the description of shapes. Owens (2002) found that precise mathematics terminology can readily be adopted by young children rather than using imprecise terminology and then changing this as they progress through schooling. Appropriate terminology should be used from the Foundation year onwards.

The Representation, Structure and Framing of the Strand

To a large extent, there are no content-based surprises in this strand—in the sense that most of the sub-strands identified in the document have been traditionally present in most other state syllabi. Thus, the content that framed state documents has been encapsulated in this national strand. Indeed, there does not appear to be any content taken out of the measurement and geometry strand and moved into one of the other two strands (whereas *chance* understandings, for example, have moved from Number strands of state curricula and embedded within the Statistics and Probability strand of the Australian Curriculum).

A fundamental change in framing this strand of the curriculum has been the inclusion of the term Geometry to describe concepts and understandings that are connected and related to measurement concepts and understandings. In previous state syllabi, geometry (or Space as it was commonly referred to in most documents) was not incorporated into the measurement strand. Furthermore, the term geometry was rarely used to describe mathematics understandings in primary school settings (only named in the New South Wales and South Australian syllabus documents). In the Australian Curriculum, the term geometry is used to describe content in both primary and high school situations. Although many teachers could bring these changes down to semantics; noteworthy, is the extensive history and debate regarding the use of this term in mathematics curricula. For example, Riemann (cited in Millman, 1977) maintained that geometry and space are distinct fields within mathematics and that geometry was founded on proofs and axioms. By contrast, space was transformational in nature and design. Clements, Grimson and Ellerton (1989), in a chapter which outlined historical changes in mathematics curricula in Australia, noted that more pure forms of Euclidean geometry had been replaced with transformational geometry under the (New) mathematics movement of the 1970s. From our perspective, the name change to Geometry does not come with a shift in intent or practice in the new Australian Curriculum.

Not only has the term space (and its stand-alone position as a content strand) been removed from the mathematics curriculum, the underlying emphasis on spatial and visual reasoning has been somewhat neglected. We argue that notions of spatial and visual reasoning are not only essential ingredients of mathematical thinking and processing (Diezmann, Lowrie, Sugars, & Logan, 2009; Lowrie & Diezmann, 2009; Owens & Outhred, 2008; Presmeg, 2008) but are increasingly important in a digital age where tools provide increased flexibility to represent mathematics ideas in graphical forms (Lowrie & Logan, 2007). As Battista (2007) highlighted in his chapter on the development of geometric and spatial thinking in *The second handbook of research on mathematics teaching and learning*, spatial reasoning is both a critical and integrated aspect of the measurement and geometry field. The development of geometric thinking relies on the production of drawings, diagrams and spatial reasoning. Battista (2007, p. 844) claimed "in geometric thought, one reasons *about* objects; one reasons *with* representations". In fact, there is no reference to spatial or visual reasoning in the entire document. The lack of attention afforded spatial reasoning in the curriculum is compounded by the fact that no indirect mention of such processing is framed within the four proficiency strands. For example, there is no mention of "drawing a diagram", "imagining in your mind's eye", or any intent to promote reasoning which encourages students to manipulate or move objects within an internal, visual, space. Such processing is accepted as an essential aspect of mathematics reasoning. Without such reasoning, the depth of understanding within this mathematics strand is lost. Therefore, the "signposting" (for teachers) that spatial and visual reasoning is critical to this strand has been removed from both content and mathematical proficiencies.

The other signposting change involves the placement of *Measurement* and *Geometry* within the same strand. The Australian Curriculum (ACARA, 2011) describes this change:

> Measurement and geometry are presented together to emphasise their interconnections, enhancing their practical relevance. Students develop increasing sophistication in their understanding of size, shape, relative position and movement of two-dimensional figures in the plane and three-dimensional objects in space. They investigate properties and use their understanding of these properties to define, compare and construct figures and objects. They learn to develop geometric arguments. They make meaningful measurements of quantities, choosing appropriate metric units of measurement. They understand connections between units and calculate derived measures such as area, speed and density. (p. 2)

In most other (state) syllabus documents, measurement was afforded its own placeholder particularly in primary school curricula. The representation and framing of a document which integrates the conceptual foundations of measurement and geometry should be applauded since rich learning opportunities

and deep thinking can occur. As with all of the sub-strands within this strand, the sequencing and development of concepts and understandings seem both logical and appropriate. We welcome this connectivity however, measurement concepts and understandings, in particular, have a narrow focus. As mentioned in the previous section, we are somewhat disappointed with the fact that the representation and framing of the document is displayed in a piecemeal manner. So many concepts and understandings are addressed within a given Year level (e.g., measure and compare length, mass, capacity and time with relationships between time units) without any clear modelling or direction. For example, would it not be best to actually compare relationships between mass units and relationships between capacity units as well as relationships between time units? If such learning opportunities are not promoted, engagement with this strand in the curriculum will be limited to procedural knowledge, the development of formulae, and conversion procedures between units of measure. As many experts have argued, relatively sophisticated measurement and geometry understandings can be developed from an early age provided students are afforded opportunities to explore, interrogate and derive meaning from realistic and challenging experiences (Clements & Sarama, 2004; MacDonald, 2010). The intent of this strand is admirable, however unless much more explicit connections between concepts are presented, this strand of the curriculum will be fragmented.

Assessment Practices: Backward Mapping from National Assessment

Although we do not advocate for national testing—in fact, we find it both problematic and limiting—we acknowledge that high stakes testing has an influence on teaching and pedagogical practices. Furthermore, the adoption of Australian Curriculum may well be influenced by how well it aligns to the type of content currently being presented in the National Assessment Program: Literacy and Numeracy (NAPLAN). The introduction of the NAPLAN has provided a control mechanism for standardising what is being taught within the respective state syllabi. The Australian Curriculum further reinforces this consistency and yet the public accountability that surrounds the NAPLAN will remain influential with or without a national curriculum. Researchers (including Reid, 2010 and Wyatt-Smith & Klenowski, 2010) have argued that the formation of the national curriculum has been done with little recognition of the assessment evidence used to inform the advancement and use of achievement standards—possibly because most states have been wary of the extent to which the content of the national document mirrors that of their state document. Since little is known about "the nature and extent of assessment evidence" that teachers will be required to collect and analyse (Wyatt-Smith & Klenowski, 2010, p. 38) we can only assume that the NAPLAN will drive assessment practices and state standards. It is vital that teachers are familiar and comfortable with the alignment of achievement standards to assessments of the Australian Curriculum (Wyatt-Smith & Klenowski, 2010).

Elsewhere, our colleagues have identified the high proportion of graphics tasks in national mathematics assessment items (Diezmann, 2008; Greenlees, 2011). Many of these items require high levels of spatial reasoning (Lowrie & Diezmann, 2009)—skills that are often developed within this strand of the curriculum. We undertook an analysis of the mathematics items in the first three years of the NAPLAN (2008 to 2010). A high proportion of these items required specific measurement- and geometry-content understandings. If we consider the 2008 data initially, in the Year 3 instrument, for example, 43% of the questions (15/35 items) had a measurement or geometry base. This high proportion was consistent across the other three tests in that year with 53%, 59% and 50% for Years 5, 7 (combined tests) and 9 (combined tests) respectively. We also analysed the questions within the categories of measurement and geometry. Across each year level, there were more geometry-based items than measurement items with questions that required an understanding of both categories (i.e., measurement and geometry) not required until Year 7 in the 2008 tests. In fact, the Year 7 test required students to link conceptual understandings from both categories, or across other strands, on 20% (13/64 items) of those items identified within this strand. More connected items appear in the 2009 and 2010 tests, with evidence of integrated concepts at Year 5.

Measurement and geometry concepts certainly feature prominently in what is being assessed at a national level. However, it could be indicative of how these topics are being taught presently that there are a minimal number of items in which integrated concepts are being assessed. It will be interesting to see how the national assessment will reflect the new curriculum once implemented.

Discussion and Conclusions

One of the most positive aspects of this strand is the fact that measurement and geometry are seen as interconnected however teachers need to be supported in effectively utilising this strength. The Australian curriculum will not have an impact on teaching and learning unless there is a significant and sustained professional development program incorporated into implementation of the curriculum. Moreover, the fact that measurement and geometry (formally space) are now embedded within the same strand provides great potential for teachers to provide learning opportunities that are rich and conceptually connected.

A number of conclusions emerged from this chapter which have direct impact on teaching and pedagogical practices and the professional development that needs to surround the implementation of the curriculum.

- It was pleasing to see some connectivity of concepts and understandings within sub-strands and across strands. We advocate that more could be included and that learning experiences should include the simultaneous presentation of concepts (e.g., perimeter-area; area-volume; volume-capacity) in order to provide opportunities to engage in rich, open-ended investigations.
- Further learning opportunities should be presented within the Curriculum so that students who are conceptually ready to engage more deeply with mathematics understandings can do so. A clearer scaffold of content should be provided so that teachers are better equipped to move students towards more sophisticated conceptual understandings.
- Correct measurement and geometry terminology should be introduced immediately to the curriculum rather than using everyday words which become obsolete as students progress through the curriculum.
- The move to combine measurement and geometry in one strand provides teachers with opportunities to emphasis interconnections within and between concepts and this heightens the practical relevance of this strand. It is important, however, that teachers are provided with focused and sustained professional development in order to ensure such connections are made.
- It is disappointing that spatial and visual reasoning has not been afforded any prominence in the Curriculum. Given the types of skills students require in a technology age, there is no better place to reinforce such processing than in this strand.
- It is evident that measurement and geometry concepts are well represented in national assessment instruments. The high proportion of items which relate to these concepts will inevitably ensure the strand has high prominence in the foreseeable future.

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Chapter 5
Preparing Students for Decision-Making in the 21st Century – Statistics and Probability in The Australian Curriculum: Mathematics

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Denise Neal

The place of statistics in *The Australian Curriculum: Mathematics* has been controversial from several perspectives. These perspectives are considered throughout the chapter. Discussion includes a comparison of the content of the Statistics and Probability strand with the model of a statistical investigation leading to beginning inference. Relevant research that underlies the current or alternative content is presented. An important focus of the chapter is the relevance of context to the study of statistics and probability, along with the need for numeracy across the entire curriculum and the contribution of statistical literacy to this goal. Interviews with classroom teachers are used to document implementation of Statistics and Probability in the classroom and to gain suggestions on changes to the curriculum and professional learning required to assist further implementation. Work samples illustrate some of the implementation that has taken place. Synthesizing these aspects results in recommendations for further research in classrooms with teachers and students in order to obtain outcomes that will assist students in the 21st century to make meaningful decisions in relation to data and risk.

The Mathematics Curriculum

Following the lead in the United States (National Council of Teachers of Mathematics (NCTM), 1989), Chance and Data became one of the components of *A National Statement on Mathematics for Australian Schools* in 1991 (Australian Education Council). This *National Statement*, although not creating a "national curriculum" for the country, influenced most state curriculum documents for nearly 20 years. When finally *The Australian Curriculum: Mathematics* first appeared in 2010, Chance and Data had transformed into Statistics and Probability. This change was itself somewhat contentious due to the fear of intimidating primary teachers with the more sophisticated terms. The terminology move, however, reflected the perceived

In B. Atweh, M. Goos, R. Jorgensen & D. Siemon, (Eds.). (2012). Engaging the Australian National Curriculum: Mathematics – Perspectives from the Field. Online Publication: Mathematics Education Research Group of Australasia pp. 89-113.

need to link concepts from preschool to Year 12, where the latter two terms were more appropriate.

In 1991, there was virtually no research available on the development of school students' understanding of concepts associated with statistics and very little related to probability. Curriculum decisions were hence made based on advice from tertiary statisticians, with very much of a top-down approach reflecting the needs of introductory units at university. In the intervening years, much research was carried out both in Australia and overseas that could have informed the writing of the new curriculum from a bottom-up developmental perspective. Some of this research was considered, whereas other aspects were ignored. Also apparently given slight attention were other nations' recent curricula (e.g., NCTM, 2006; Franklin et al., 2007; and Ministry of Education, 2007, 2009). What is presented in the new curriculum is, hence, a mixture of good suggestions and questionable propositions, constrained by pressure from the other two strands of Mathematics, Number and Algebra and Measurement and Geometry. Given that some mathematicians continue to hold the view that statistics should not be in the mathematics curriculum at all (e.g., Dean, 2010), it is important to work consciously to build on what is there to create a meaningful component of the curriculum that can lead to the critical statistical literacy required by students as they become citizens of Australia in the 21st century.

Although it is possible to argue that aspects of theoretical probability are part of mathematics based, for example, on calculating permutations and combinations to produce probabilities, statistics is arguably different in its non-deterministic nature, which demands special consideration. "Doing statistics" is about carrying out investigations that have more than one step and that encompass the stages shown in Figure 1.

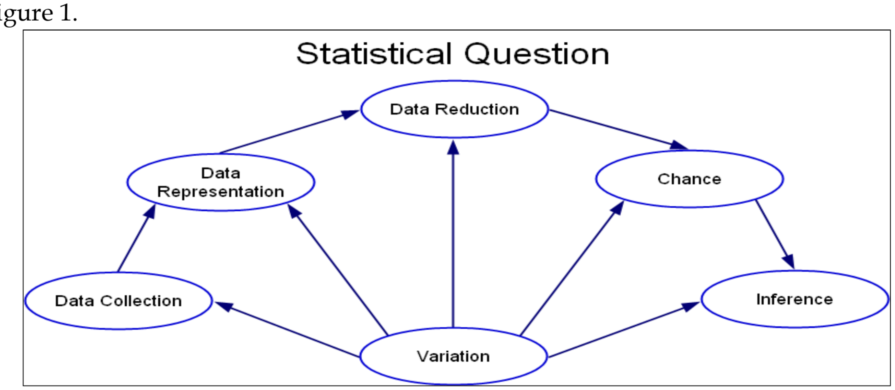


Figure 1. Model for a statistical investigation (Watson, 2009).

Understanding statistics is not just about calculating a mean or drawing a graph; it is about considering all parts of the model in Figure 1, drawing a conclusion to an investigation of a question based on evidence from data in a context, acknowledging variation and ultimately the uncertainty of the conclusion. To do all of this in every year, with ever-increasing complexity of data sets, context and available techniques, would be desirable but space in the curriculum puts constraints on what can be done. It is, hence, necessary to consider the components of the model in Figure 1 and explore the curriculum to find them.

A Statistical Question is the starting point of any investigation and immediately raises the issue of context and cross-curriculum links. Because there is no statistics without a problem from another field (Rao, 1975), immediately this part of the mathematics curriculum lends itself to satisfying the "numeracy across the curriculum" requirements of the National Curriculum Board (NCB, 2009). This link to other subjects may be one of the negatives that pure mathematicians see in a curriculum that they would like to see focused more specifically on mathematical theory and techniques. The Statistics and Probability component of *The Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2011c) begins with asking questions in the Foundation Year, with more complex scenarios being introduced over the years. Although the word "context" is not explicitly used until Year 5, the questioning suggested before then includes contexts such as hair colour, birdlife in the playground, popular breakfast cereals, and examples in the media. By Year 7 the descriptor, "identify and investigate issues involving continuous or large count data collected from primary and secondary sources" (p. 37), implies that context is involved and the elaborations provide examples. Further in Year 7 the elaboration for mean, median, and mode includes "linking them to real life." Working with context meaningfully in relation to fundamental statistical concepts, however, may involve both connection and disconnection at various stages of an investigation, as illustrated by the intervention of Pfannkuch (2011) in a Year 10 classroom.

The underpinning concept for the entire field of statistics is that of Variation (Moore, 1990) and although the term is not defined in the glossary, it is explicitly mentioned in Year 3 in relation to Chance. Why it is first linked to Chance and not to Data Representation and Interpretation is a mystery, but at least it is there and teachers can discuss variation across all phases of an investigation, where they begin with data from random generators, such as dice, or from other sources in real-world contexts. In Year 4, however, variability is one of the criteria used for evaluating the effectiveness of different data displays. Later in Year 8, a descriptor asks students to "explore the variation of means and proportions in representative data" (p. 40). This statement is important but it is unfortunate there are no elaborations to help

teachers interpret it. The type of elaborations that are appropriate are found in Watson (2005) for students from age 6 to 15 and in Shaughnessy (2006) especially related to studying graphs to characterise distributions. Hopefully by high school, thinking about variation will accompany every investigation.

The first component of an investigation (Figure 1) is Data Collection and from the Foundation Year students are "collecting information." In Year 1, they "gather responses" and in Year 2, "gather data." By Year 3, students "conduct chance experiments … plan methods of data collection and recording … and collect data …" In Year 4, survey questions are introduced as a data collection method. Year 5 contains the first explicit mention of collecting "categorical or numerical data". Although the elaboration of this descriptor in Year 5 describes a sampling situation, the actual term "sample" is not introduced until an elaboration in Year 6. The associated descriptor asks students to "interpret secondary data presented in digital media and elsewhere" (p. 33), an important inclusion, although the potential for using such data is clear in earlier years where context could lead to exploring data sets from outside of the classroom. As well at Year 6 the elaboration of this descriptor introduces the critical thinking necessary to question aspects of data collection and possible biases. Large data sets are mentioned in Year 7 and the implications of collecting representative data are made explicit in Year 8. In Year 9, the descriptor tying surveys from digital media to estimated population means and medians appears misplaced under Chance rather than Data Representation and Interpretation. It is not until this level that collecting data involving "at least one numerical and at least one categorical variable" (p. 43), including secondary sources, is mentioned. Research has shown (e.g., Watson & Donne, 2009) that students as young as Year 5, and certainly in Year 7, can handle these two types of variable, for example using class data and data from the Australian Bureau of Statistics website Census@School (www.abs.gov.au).

Data Representation is the second stage in statistical investigation (Figure 1) and is mentioned at every year level in the curriculum, becoming more complex with increasing years. There are many types of data representation and it was difficult for the writers to select ones that should be explicitly mentioned in the curriculum and at what year. Because technology is allowing much more variety in the types of graphs available in the media and hence in other subject areas (e.g., Wall & Benson, 2009), what the curriculum needs to encourage is flexibility in thinking about and interpreting graphical forms. The acknowledgement of the use of digital technology to create graphs from Year 3 is encouraging and the technology should be employed at every year level from there. Some of the specific suggestions of graph types are controversial, because they appear to underestimate the ability of students to handle the task. Explicit mention of graph types in the descriptors includes picture graphs in Year 2, picture graphs and simple column graphs in Year 3, the same in Year 4 with many-to-one representation, column graphs and dot plots in Year 5, side-by-side column graphs for two categorical variables in Year 6, stem-and-leaf plots in Year 7, back-to-back stem-and-leaf plots and histograms in Year 9, and box plots in Year 10. The most concerning omission is the scatterplot, which is not mentioned until an elaboration at Year 10. Research has shown that students can create (with technology) and interpret scatterplots as young as Year 6 (Watson & Donne, 2009; Fitzallen & Watson, 2010). In dealing with two numerical variables, a scatterplot is a natural representation.

Data Reduction is the next stage in an Investigation, usually associated at the school level with measures of central tendency (i.e., average) and spread (i.e., variation). As noted earlier, variation is considered generally throughout the years. The calculation and interpretation of the mean, median and mode, to measure central tendency, are not introduced until Year 7. The idea of "most" (or mode) is accessible to children as soon as they can count and compare numbers and should be noted much earlier. Working with the middle value of an ordered data set, such as the heights of students lined up at the front of the classroom, is also accessible to much younger children and builds intuitions about centre and representativeness. The measures are applied in exploring samples and populations in Year 8. This is an important point because although measures of average have been a focus of research for many years (e.g., Mokros & Russell, 1995), it has also been shown that often students do not employ the measures naturally when completing open-ended tasks, for example comparing two groups (e.g., Watson & Moritz, 1999). The range, as a measure of spread, is first mentioned in Year 7, outliers are noted in Year 8, and centre and spread are used to compare data displays in Year 9. In Year 10, the box plot provides a visual representation of both the centre and the "middle 50% of the data," as well as the spread. The importance of box plots is seen in the research of Pfannkuch (2006) exploring a teacher's struggles with comparing box plots.

Chance is separated from Data Handling and Interpretation in the curriculum and there is very little effort to link the two, which is unfortunate. There should be an intuitive recognition that chance and probability are involved in determining the confidence with which decisions are made in statistics. Much time is spent in the early years talking intuitively about the likelihood of events happening. After initial work with language, which progresses very little up to Year 4, the treatment of chance is very mathematical. Having introduced the frequency approach to probability in Year 6, sample spaces appear in Year 7. Explicit mention of contrasting the two approaches would be useful, as would acknowledgement that often chances are determined in a totally subjective manner. From the introduction of two-way tables in Year 8 through to Year 10, the descriptors are not explicit

enough in making clear to teachers and students what is meant. It appears that in trying to avoid language such as "conjunction" and "conditional," a vagueness associated with Venn diagrams and two-way table makes it difficult to understand the explicit intentions of the curriculum. Further elaborations or worked examples for problems exemplifying the descriptors are needed. Over the years there has been considerable research on students' understanding of probability (e.g., Jones, 2006) and it is unfortunate that outcomes were not used to inform the curriculum. Given the statistical tools that students have acquired by Year 10 they cannot determine specific numerical probabilities associated with the likelihood of a decision, for example about whether two populations are "different." Using box plots, however, and criteria suggested by Wild, Pfannkuch, Regan and Horton (2011), they could make informal likelihood statements about the populations represented by samples of different sizes.

Inference, the last stage in a statistical investigation, is acknowledged to a limited extent in the curriculum with the Interpretation part of Data Representation and Interpretation. The word interpret is used frequently from Year 2 to Year 10. Its use in later years should imply a sense of "beginning inference" (Watson, 2006) or "informal inference" (Makar & Rubin, 2009), where *evidence* is used to make a *generalisation* beyond the data with an acknowledgement of *uncertainty*. Although evidence is implicit in referring to data, it needs to be made explicit, as does uncertainty throughout and generalisation in later years. From the beginning, data in a context provide the evidence that students can interpret, for example, "the most popular fruit in our class is the banana." To create awareness of uncertainty, teachers can ask, "are we certain that banana will be the most popular fruit tomorrow?" To create awareness of generalisation beyond the classroom, teachers can ask, "will banana be the most popular fruit for our whole school?" or "would banana be the most popular fruit in Australia?" This sample-population relationship is alluded to in elaborations in Years 10 and 10A, but it is not obvious that there are opportunities for appropriate intuitions to be developed across the years of schooling. Over 30 years ago, Kissane (1978) illustrated the feasibility of addressing "intuitive inference" in the school curriculum. Recent research (e.g., Makar, Bakker, & Ben-Zvi, 2011) suggests many avenues for developing the reasoning behind informal statistical inference that can be introduced across the years before Year 10. Recognising the importance of conflict and context provides a contrast to much of the other content within the mathematics curriculum. Another aspect of inference is prediction (e.g., Watson, 2007) and although there is a promising start in Foundation to Year 2 in making "predictions about chance events" (p. 5), this is only mentioned once again, in Year 6, and the usage does not encompass data and the opportunity to predict with uncertainty in many other contexts.

Besides the Content strands of *The Australian Curriculum: Mathematics*, there are Proficiency strands, which describe the "actions in which students can engage when learning and using the content" (p. 2). The four proficiencies – understanding, fluency, problem solving, and reasoning – are all exemplified at various points within the Statistics and Probability component of the curriculum. The "how" and "why" of understanding are seen across the stages of an investigation when data are collected, graphs drawn, and statistics calculated. Fluency is seen when students recall methods for graphing and calculating statistics and carry them out accurately and efficiently. Students demonstrate problem solving when they design investigations to answer questions, plan strategies, and check outcomes; the verbs interpret, model, and investigate are particularly relevant to Statistics and Probability. The logical thought associated with reasoning includes evaluating, explaining, inferring, justifying and generalising, all of which are involved in a complete statistical investigation. Given the contextual nature of statistical investigations the ability to transfer learning across the curriculum illustrates all four proficiencies, but particularly reasoning.

The Wider Curriculum

Turning to the other three areas of the curriculum released at the same time as Mathematics, it is of interest to explore reference to data or statistics that reflects the NCB (2009) requirement for "numeracy across the curriculum," as well as providing meaningful contexts. *The Australian Curriculum: History* (ACARA, 2011b) provides a promising start with the statement, "Students need to organise and interpret historical events and developments and this may require analyses of data to make meaning of the past, for example to understand cause and effect, and continuity and change. This requires skills in numeracy such as the ability to represent and interpret quantitative data" (p. 7).

Although not explicitly stated, the gathering of information on families past and present and communities, as well as localities up to and including Year 2 would lend itself to tallying or creating pictographs as suggested for the same years in Data Representation and Interpretation. From Year 3 the descriptor under Historical Skills, "Locate relevant information from sources provided" (p. 20), evolves into "Identify and locate relevant sources, using ICT and other methods" in Year 8 (p. 40), continuing to Year 10. Posing questions begins in Year 2 with Years 3 and 4 suggesting a "range of questions" and Year 5 moving to "identify questions to inform an historical inquiry" and "identify a range of relevant sources" (p. 24). These reflect elements seen in Figure 1 as part of a statistical investigation. An elaboration in Year 6 suggests "retrieving census data to construct arguments for and against migration" (p. 27). In Year 9, an elaboration asks students to "[graph] historical data to identify past trends and to draw conclusions about their significance" (p. 46). The Mathematics curriculum provides ample graphical forms for this enterprise. Further, in History in Year 10 students should, "[use] data from immigration records and [process] it (sic.) using ICT to identify historical trends over time" (p. 52) and "[combine] historical data from a range of sources to identify and explain the impact of World War II" (p. 52). Historical sources such as a data base on the First Fleet created by the University of Wollongong (firstfleet.uow.edu.au/index.html) can be used across year levels and have links to data handling skills in the statistics part of *The Australian Curriculum: Mathematics* (e.g., Watson, Beswick, Brown, Callingham, Muir, & Wright, 2011).

For Science, the links to statistics through "numeracy across the curriculum" are very strong, particularly in the Science Inquiry Skills strand where the five sub-strands of Questioning and predicting, Planning and conducting, Processing and analysing data and information, Evaluating, and Communicating fit very well with the model in Figure 1. Under the heading, Planning and conducting, for example, students are "Making decisions regarding how to investigate or solve a problem and carrying out an investigation, including the collection of data" (ACARA, 2011d, p. 4). Expanded under Numeracy (p. 10), these skills include "practical measurement and the collection, representation and interpretation of data from investigations … [considering] issues of uncertainty and reliability … collecting, analysing, and representing quantitative data in graphical forms, … identifying trends and patterns from numerical data and graphs … using statistical analysis of data and linear mathematical relationships to calculate and predict values." The intention is that graphical representations will be used across the years as they are developed in parallel within the mathematics curriculum, including scatterplots, linear graphs, and gradients (p. 14). It is salient that "using simple column graphs (bar graphs) with guidance from the teacher" is suggested in Year 2 (p. 22), whereas it does not explicitly occur in Mathematics until Year 3. In Years 5 and 6, "compare data with predictions and use as evidence in developing explanations" (p. 31, 34) reflects better the expectations of beginning inference than what is found in Mathematics, and describing "simple cause-and-effect relationships as shown by trends in quantitative data" (p. 34), does not appear in the Mathematics curriculum. Calculating means and ranges in Science appears in Year 9 (p. 45), a year or two after Mathematics. In Science, skills are repeated in the descriptors at later year levels with the elaborations intended to illustrate the more complex contexts for their consideration. The contexts suggested are those related to the understanding of the content sub-strands of the biological, chemical, earth and space, and physical sciences, and they vary widely. English (2009), for example suggests data modelling in relation to a class project exploring pollution in a local creek. Examples of explicit links between data handling and activities involving measurement of data, such as the number of drops of water that will fit on a five-cent coin or the percentage of weeds in a garden (Brown, Watson, & Wright, 2011), provide many opportunities for cross-curriculum classroom engagement.

Context and critical thinking are the major links between *The Australian Curriculum: English* (ACARA, 2011a) and *The Australian Curriculum: Mathematics* (ACARA, 2011c). "Numeracy can be addressed in English learning contexts across all year levels. Students select and apply … graphical [and] statistical … concepts and skills to real-world situations when they comprehend information from a range of sources and offer their ideas. When responding to or creating texts that present issues or arguments based on data, students identify, analyse and synthesise numerical information and discuss the credibility of sources and methodology" (ACARA, 2011a, pp. 10-11). Under the Literacy strand of English, the sub-strands of "Texts in context" and "Interpreting, analysing, and evaluating" provide the avenues for intersections with statistical thinking. Although there is no direct mention of numeracy- or statistics-related usage, an elaboration at Year 5 suggests "bringing subject and technical vocabulary and concept knowledge to new reading tasks" and "using research skills including … gathering and organising information … and summarising information from several sources" (p. 45). The introduction of media texts in Year 6 provides opportunity for synergies with statistical goals, also introduced in Mathematics in Year 6 (e.g., Watson, 2004). Cause-and-effect is mentioned in a descriptor at Year 7 (p. 55), following Science in Year 6 but there is no mention in Mathematics. At Year 8, the descriptor "Apply increasing knowledge of vocabulary, text structures and language features to understand the content of texts" (p. 60) could well apply to language from the Statistics and Probability strand of Mathematics. Similarly, "Interpret, analyse and evaluate how different perspectives of issue, event, situation, individuals or groups are constructed to serve specific purposes in texts" (p. 65) in Year 9 could well apply to investigations of media claims based on questionable statistics. Despite these general statements the lack of any specific mention of a context involving numeracy (or statistical literacy) is somewhat disappointing given the initial introductory statement on numeracy.

A final comment from across the curriculum arises in the preliminary *Shape* statement for Geography (ACARA, 2010, p. 8) in relation to Numeracy: "Students … count and measure, calculate statistics and interpret them, and construct and interpret graphs. In the senior secondary years, interpreting the results of statistical analyses requires an understanding of mathematical concepts such as probability." Whether this final requirement will be met from the mathematics curriculum in Years 11 and 12 is uncertain.

There are three Cross-curriculum Priorities in the Australian Curriculum, all of which, given the previous discussion, lend themselves to data analysis. At the introductory level, "Aboriginal and Torres Strait Islander histories and cultures" provide a link between History and Statistics. "Asia and Australia's engagement with Asia" can combine Geography, History and Statistics. "Sustainability" is likely to begin with Science and Statistics but also link to Geography. In all cases studying data bases can add depth and rigour to classroom investigations.

The overall General Capabilities within the Australian Curriculum are Literacy, Numeracy, Competence in ICT, Critical and creative thinking, Ethical behaviour, Personal and social competence, and Intercultural understanding. The concepts of variation and expectation, along with acknowledged uncertainty, the foundation of the Statistics and Probability curriculum, are readily enlarged to underpin the complexities of the general capabilities, particularly the final three. The first four are encompassed in the earlier discussion in this chapter, with "statistical literacy" listed as one of the six mathematical skills expected to be developed by students related to the Numeracy competency. In terms of ICT it is somewhat disappointing to note that although digital technology is mentioned periodically in descriptors in the Statistics and Probability section of the mathematics curriculum, statistical software does not rate a mention in the generic list of digital technologies, which includes "spreadsheets, dynamic geometry software, and computer algebra software [to] engage students and promote understanding of key concepts" (ACARA, 2011c, p. 9). Further the Achievement Standards at the end of each year level do not mention achievement related to technology in any form across the entire F-10 curriculum.

Reaction to the Curriculum

In the first stages of the rollout of *The Australian Curriculum: Mathematics*, teacher feedback was sought from 17 staff of a primary school in Tasmania and 5 secondary teachers who were leaders in their schools or consultants. When asked the areas in which they had confidence the majority of primary teachers were happy with basic chance/probability or the language associated with it (as covered in the early years of the curriculum), with the basic notions of data collection, and with graphing. A few teachers mentioned tables and picture graphs, as well as interpreting data. The picture reflects confidence in the basic skills of chance and data as reflected in previous curriculum documents but not the subtle or higher level expectations as students move in to middle school (e.g., Department of Education, 2007). For secondary teachers who were qualified in mathematics, teachers had confidence in all content areas. For less qualified teachers, usually in rural schools, despite knowing the "basics," there was concern about struggling with the curriculum for students at specific year levels. Without the separation of lower, middle, and higher expectations for different students within a year, some teachers expressed lack of confidence with respect to both teaching and assessment, a difficulty not experienced with the standards in the Tasmanian curriculum. The "new" language was also seen as an issue for some teachers in the middle school.

When asked to name the areas where the primary teachers felt they needed more support, fifteen different aspects of the curriculum, with specific content, were noted. These included chance phrases, where a teacher desired "certainty" about the interpretation of words like "maybe" and "possible"; the linking of probability to fractions, decimals, and percents; comparing experimental and expected frequencies across (random) trials; interpreting the descriptors under Chance in the curriculum document; formal ways of displaying data in the early years; working with secondary data (from sources outside the classroom); different ways to collect data; recording data in various ways; relating data displays to the sample size; use of the media; an overall understanding of the terminology; and the need for cross-curriculum links. More generally the most frequent request by primary teachers was for access to and help with digital technology to represent data, including access to data bases. For the secondary teachers, professional learning needs across schools and teachers were varied but included the following: access to resources to move teachers away from text books; exposure to "good" large data sets; introduction and practise on software; data representation and interpretation in the senior years; designing tasks for specific proficiencies; finding and developing rich tasks; understanding correlation and its relationship to r-squared; and developing chance activities beyond rolling dice.

When asked for their concerns about the curriculum, teachers were generally happy with the Statistics and Probability component, although they felt there had been a "narrowing down" from the existing Tasmanian curriculum; for example, there was not the same emphasis on variation and expectation as in the Tasmanian document. They felt there was a need for more elaborations for the content descriptors, more like what is promised in Scootle <www.scootle.edu.au>, and directly linked to content. Concern about defining content for specific year levels was expressed by several teachers but others said they only viewed the content as a "progression," quoting one section of the curriculum acknowledging that teachers first should "identify current levels of learning and achievement and then select the most appropriate content (possibly from across several year levels)" (ACARA, 2011c, p. 12).

Specific improvements to the curriculum suggested by the secondary teachers included moving box plots and stem-and-leaf plots to earlier years; making clear the distinction between discrete and continuous variables in Year 7; taking a practical

approach to probability in Year 7 linking it to statistics via collecting data from chance activities to use for statistical investigations; improving the treatment of scatterplots (much earlier than Year 10) and incorporating technology to include lines of best fit; and including cumulative frequency, and a contrast of percentage and percentile, as precursors to studying box plots. Concern was also expressed that the expectation for time allocation for *The Australian Curriculum: Mathematics* is 5 hours/week in Primary and 4 hours/week in Secondary. It was claimed that schools that value mathematics may currently allocate only 3 hours/week to the subject and many allocate much less; this is not closely monitored, at least in state schools. With this issue in mind the curriculum was seen as "aspirational" with Year 6 outcomes perhaps being realistic at the end of Year 7.

When teachers were asked what contexts they were aware of or could suggest that would link to Statistics and Probability from across the curriculum, teachers mentioned Science, with some specifically highlighting the weather and, Literacy in relation to genre, writing, vocabulary, claims made, and favourite books. Geography, History and Physical Education were noted, as was using data collected from the class, for example transport to school, birthdays and family data. One teacher said it was important to use the media in relation to advertising found there using data to persuade. Teachers as a group were hence aware of potential contexts but from their previous responses it appears they lack well developed resources and technology to use them in their classrooms.

Work Samples

Some of the teachers who were interviewed provided work samples from units of work completed by students in their schools. The first example is from a Year 1/2 class where the students recorded the suburbs in which they lived. After class discussion, one of the representations created was a stacked dot plot using cubes of different colours. A photo was taken of the plot and used to create a sheet for the students to record their stories about the data they had collected. The range of responses was large and shows the potential for such an activity at the year level. Four examples of student work are shown in Figure 2.

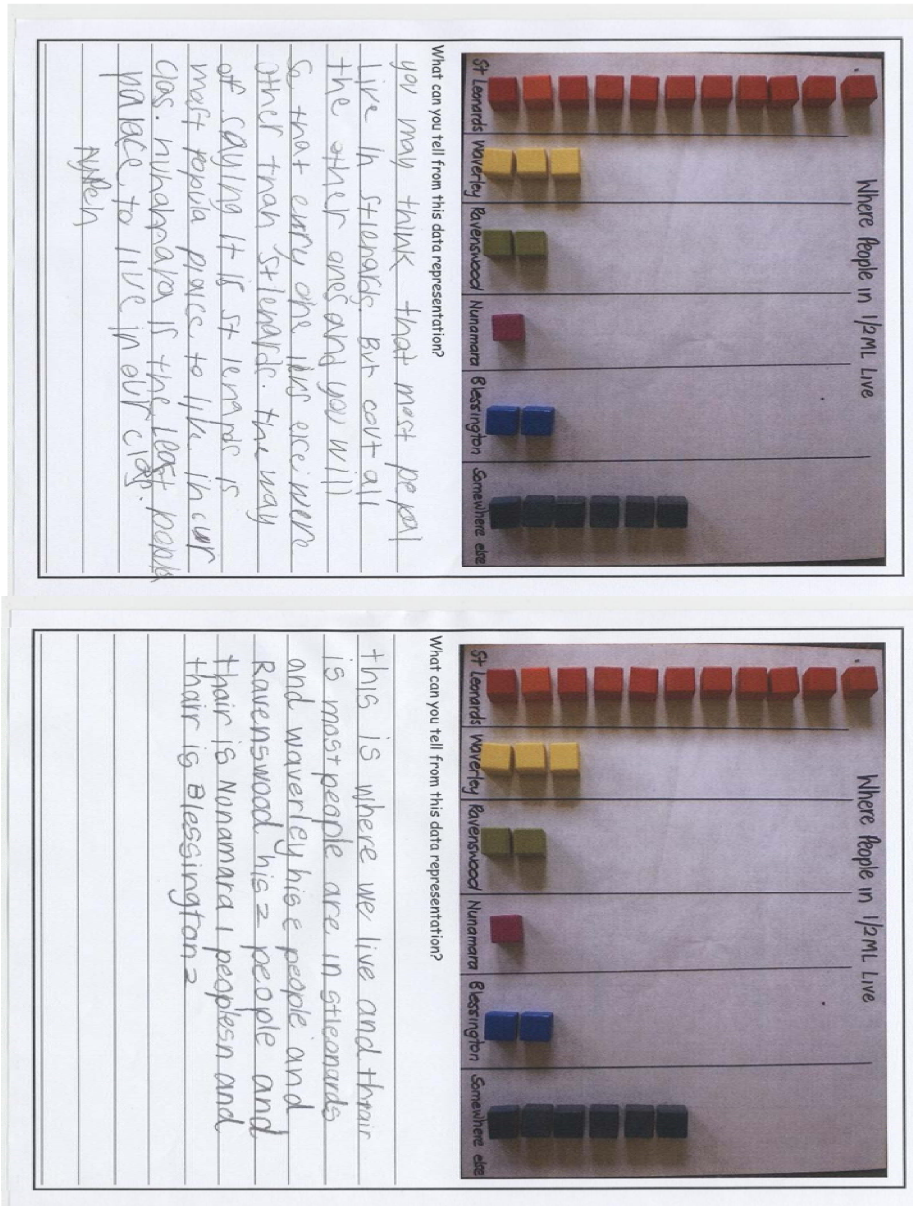


Figure 2. Year 1/2 student work.

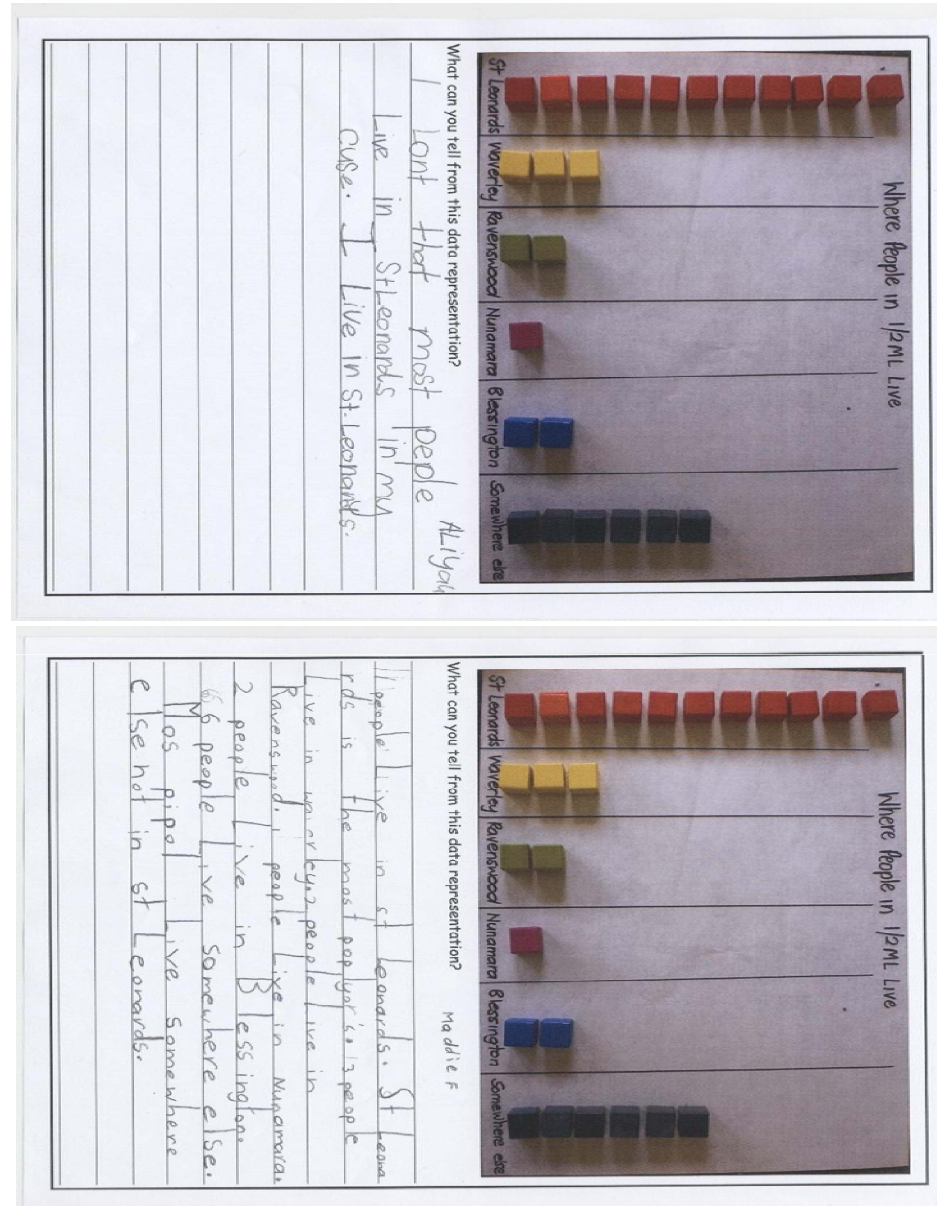


Figure 2. Year 1/2 student work (continued).

What is particularly interesting in the responses in Figure 2 is the reflection in the discussion about the "most popular" suburb and the total number of children who live in other suburbs. It appears clear that at least several students had picked up this distinction and could describe it. These two sets, "St Leonards" and "other suburbs" are disjoint and complementary with respect to the entire class. Although "Identify complementary events" is included as a descriptor in Year 8 for Chance, following the descriptor "Identify everyday events where one cannot happen if the other happens" in Year 4 (ACARA, 2011c, pp. 26, 40), it is clear that the foundation for understanding such concepts can be laid much earlier when children are discussing data related to themselves and their environment.

Figure 3 contains the "poster" prepared by a Year 4/5 class based on an investigation of the length of their first names, an investigation suggested by the Australian Association of Mathematics Teachers several years ago for National Mathematics Week. It illustrates the nature of a statistical investigation that is possible in the middle years, including much more than creating a graph, and satisfies the content descriptors for Data Representation and Interpretation in Year 5.

At the high school level one of the teachers explored data collected from the Australian War Memorial database to work with his mathematics class on data investigations before Anzac Day (P. Tabart, personal communication, 19/4/2011). This activity involved the use of the software *TinkerPlots* (Konold & Miller, 2005) with classes in Years 8 and 10. The data set was very motivating in an all-boys school and the reaction of the class was very positive. Figure 4 shows the type of data collected for 45 months of World War I that were held in data cards in *TinkerPlots*, some of the questions that the students considered, and three of the plots that were created for or by the class. The outcomes were consistent with expectations across Years 8 to 10, including working with representative data, investigating an international issue, using displays to investigate effects, using authentic data to construct scatterplots and draw conclusions, and use mean, median and range to interpret numerical data sets (ACARA, 2011c, pp. 40, 43, 46). This data set and the work in a mathematics classroom would provide an excellent adjunct to a unit in History at the same year level. Similar data sets exist or can be created for data from the Titanic, Australian explorers, or early Australian convicts (e.g., Watson et al., 2011).

Figure 3. Year 4/5 student work.

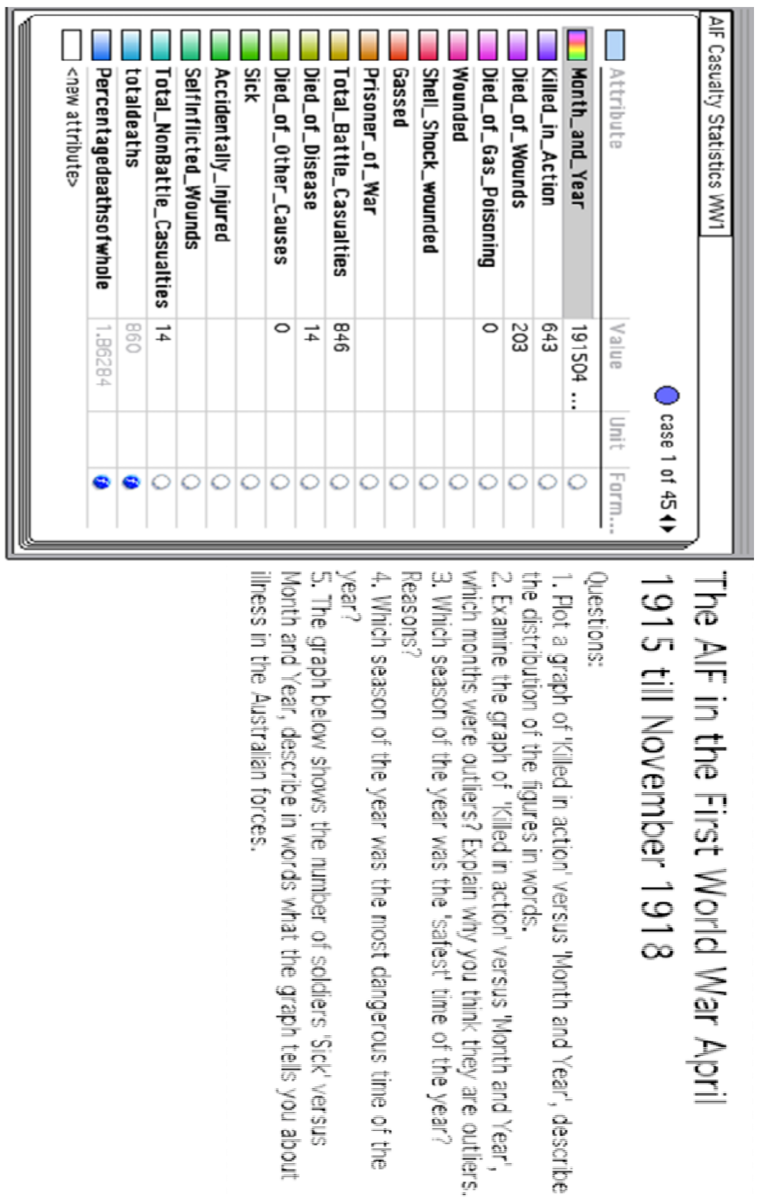


Figure 4. High school activities linked to Anzac Day.

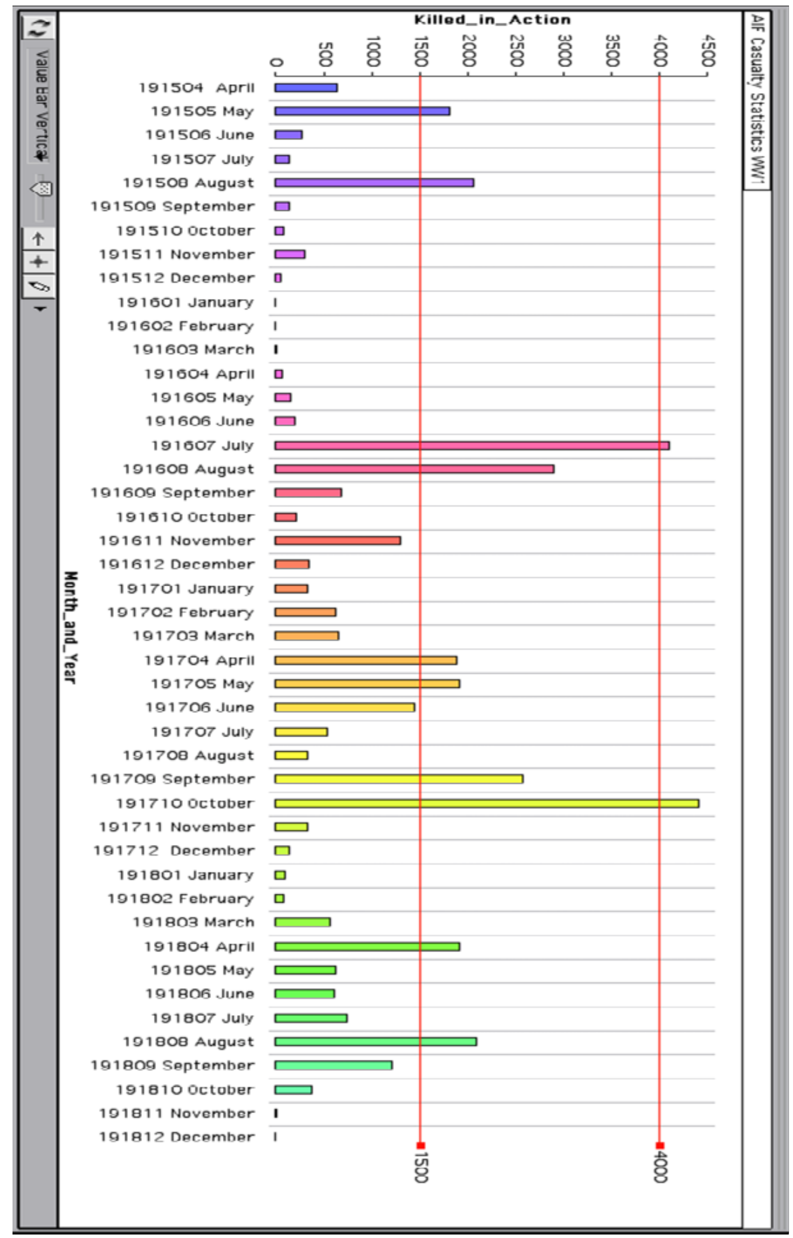


Figure 4. High school activities linked to Anzac Day (continued).

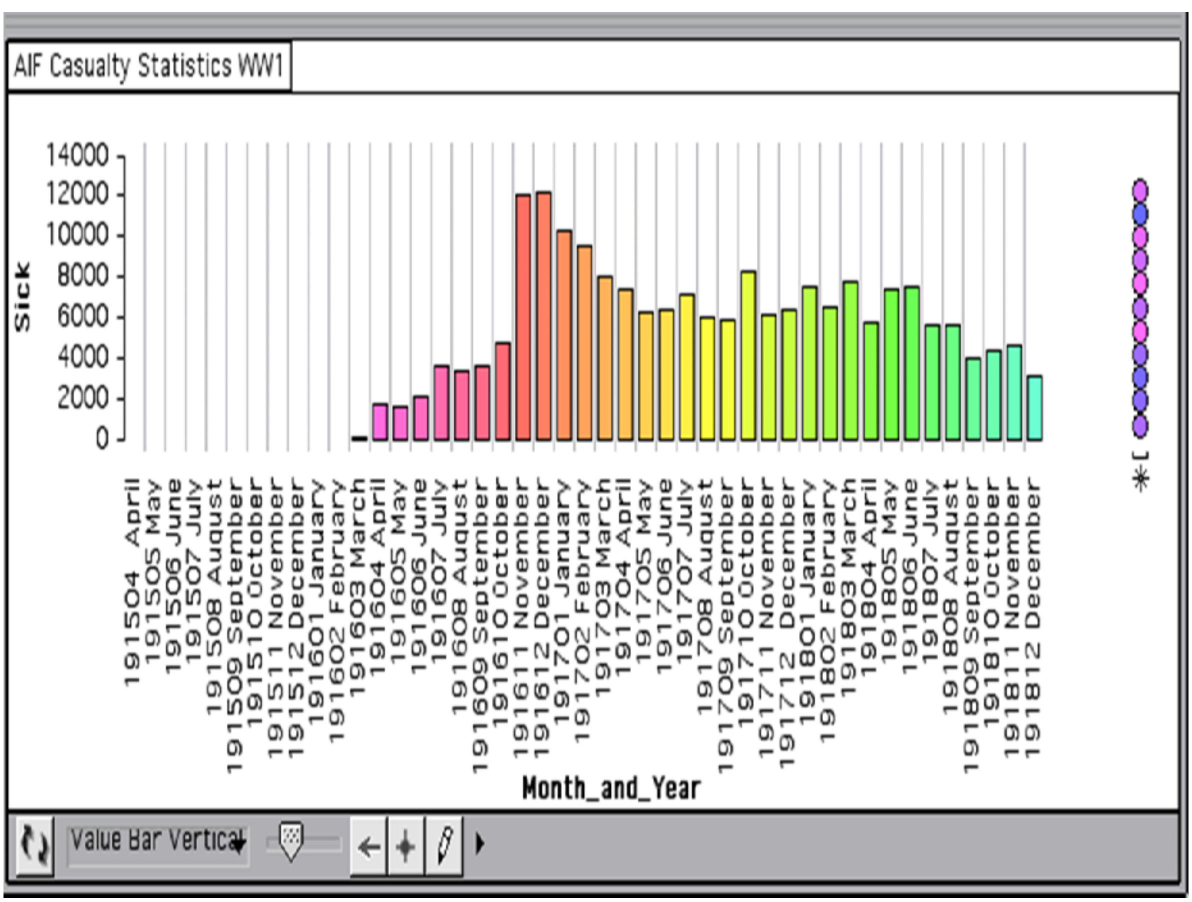
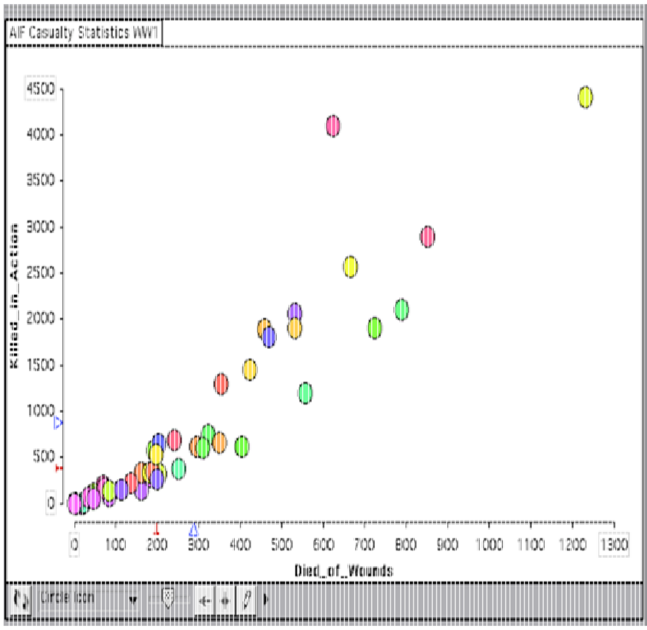


Figure 4. High school activities linked to Anzac Day (continued).



Problems to answer

1. Create a plot of Killed in Action. Use this to determine the following:
a) The mean number of soldiers killed in action per month.
b) The median number of soldiers killed in action per month.

2. Create a plot of Gassed against Month and Year. What conjectures can you make by looking at this graph? Give reasons.

3. Create two plots; one of self inflicted wounds versus month and year and the second plot of totaldeaths versus month and year. Examine both graphs side by side. What conjectures can you make in relation to the occurrance of self-inflicted wounds? Give reasons.

Figure 4. High school activities linked to Anzac Day (continued).

Implications and Further Research

As was seen at the beginning of this chapter, the components of a statistical inquiry are covered across the years of the K-10 mathematics curriculum. It is unfortunate that an effort is not made to reinforce their integrated relationship with each other at all levels of schooling. Teachers who are aware of the connections, however, can discuss and speculate with students each year, as was illustrated by the Year 1/2 class that provided the work samples in Figure 2. This is where professional learning can be of assistance, taking the elements of content with which teachers are familiar at each level and combining them to give a big picture of what statistics is about when considering a problem based in an authentic context. For primary teachers this can happen readily as most teachers cover the entire curriculum in their classrooms. For secondary teachers there is an urgent need to provide combined professional learning for teachers from differing curriculum areas, including mathematics. Teachers from other areas may need to increase their own skills in data handling and informal inference, likely supplemented by use of software, whereas mathematics teachers may need to increase their understanding of how project work and investigations are carried out in meaningful contexts in other areas. School time-tabling may be an issue for implementation of joint lessons but it was sorted out in one of the schools where a teacher was interviewed for this chapter. The three cross-curriculum priorities mentioned earlier related to Aboriginal and Torres Strait Islanders, Asia, and Sustainability all offer great potential for linking mathematics, through statistics, to other subject areas such as History, Geography, and Science.

Although the Content strands have been the focus of much of the discussion about the *Australian Curriculum: Mathematics* (ACARA, 2011c), the Proficiency strands provide the mechanism for implementation of the curriculum. The proficiencies are likely to be the foundation of meaningful professional learning for teachers, assisting with the development of pedagogies to aid understanding, fluency, problem solving, and reasoning. Teachers need to be aware of these four aspects of their own thinking before creating learning experiences for their students. As specifically stated in the curriculum, Problem solving for example is illustrated by "interpreting sets of data collected through chance experiments" (Year 7, p. 34), "interpreting data using two-way tables" (Year 8, p. 38), and "collecting data from secondary sources to investigate an issue" (Year 9, p. 41). Significantly, reasoning includes "inferring from the results of experiments" (Year 6, p. 30), "making inferences about data" (Year 8, p. 38), and "evaluating media reports and using statistical knowledge to draw conclusions" (Year 9, p. 41). The reference to "inferring," to "inferences," and to "drawing conclusions" in the Proficiencies, when the word inference does not appear in the Content descriptors or elaborations,

points to the need for professional learning for teachers based around the development of informal inference across the schools years (Makar, Bakker, & Ben-Zvi, 2011; Makar & Rubin, 2009) in order to enhance the meaning of "Interpretation" in the substrand heading Data Representation and Interpretation.

There is much research to be carried out in relation to the implementation of the new curriculum. Some of the questions to be answered include the following. What are the links between students' development of proportional reasoning and their understanding of box plots? How does the use of interactive software facilitate development of problem solving and reasoning for statistical investigations? What is the best way to transition between probability seen as trials with random devices and probability seen as the measure of uncertainty for statistical investigations? How can stem-and-leaf plots be used to reinforce student understanding of place value in the lower primary years, as well as providing early intuitions about the shape of data distributions? How can statistics contribute to the understanding of estimation and approximation in the Measurement and Geometry component of the curriculum by displaying and measuring variation (e.g., Watson & Wright, 2008)? Following the research of Watson (2005), how can variation and expectation be woven seamlessly throughout the curriculum to build the foundation and intuitions needed to understand formal inference in later encounters with statistics? As part of cross-curriculum work, how can students' appreciation of perceived risk as a function of hazard and outrage (Sandman, 1993) be built upon the Probability curriculum and transformed into social contexts to assist students in making safe life decisions?

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Chapter 6
Equity and the Australian Curriculum: Mathematics

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Thelma Perso

A national curriculum in Australia (The Australian Curriculum) may represent an attempt to bring a common experience to all Australian students. It is premised on a notion that shared experiences are part of the national psyche. However, in this presentation, we challenge this approach and argue that the experiences of students living in remote parts of Australia, or those whose home language is not Standard Australian English; or those whose culture is not that of mainstream urban Australia, may be particularly disadvantaged. Other priorities may be necessary for students who consistently are unable to make benchmarks in numeracy (and literacy) which are foundational to active and successful participation in Western practices. Drawing on experiences from remote work, we seek to illustrate how a national *intended* curriculum will not necessarily improve education provision for students living in remote areas, but most particularly Aboriginal and Torres Strait Islander students for whom the language of instruction is different from the home languages. A multifaceted approach to understanding equity, and by implication, exclusion, is needed, if the goals of a quality education for all Australians are to be met.

In a diverse country such as Australia, the perceived need for a national curriculum comes from many spheres. In terms of equity, it is important to consider the contexts of Australian education and how these impact on the provision of education. Also, the political landscape, geography and spread of population constrain many aspects of education provision. The diversity of populations range from first peoples who have occupied the land for tens of thousands of years, to people who are descendants from the first fleet or the many migration waves including the Chinese in the mid-nineteenth century, through to Europeans from post-war migrations. More recently they include arrivals entering the country as refugees from African and subcontinent countries. With such diversity, there are many cultures, many languages and many diverse aspirations for education and life in Australia. Providing an education that is sustainable for governments, allows pathways for all Australians to achieve a quality life, and simultaneously allows all

In B. Atweh, M. Goos, R. Jorgensen & D. Siemon, (Eds.). (2012). Engaging the Australian National Curriculum: Mathematics – Perspectives from the Field. Online Publication: Mathematics Education Research Group of Australasia pp. 115-133.

Australians access to dominant forms of knowledge is the challenge ahead of a national curriculum.

While issues around the diversity of Australia are an integral part of this chapter, we particularly draw on remote education provision. This context enables us to highlight the challenges for a national curriculum that aims to provide all students with access to high-quality schooling and reduce the effects of disadvantage including remoteness. (MCEECDYA, 2008)

In this chapter, we discuss the implications of equity and the Australian Curriculum drawing on the contrasts between the remote parts of the Northern Territory and those of the more heavily populated states of Victoria and New South Wales. Using these comparisons, we discuss the tensions around equity in terms of provision, access and cultural diversity that are inherent through a national curriculum in a diverse country such as Australia.

The Australian Curriculum specifies what all young Australians should learn as they progress through schooling. It is designed to be taught well within the overall teaching time and with the resources available to teachers and students. School authorities make decisions about the allocation of time and other resources. (ACARA, 2010) It is evident from this information that the Australian Curriculum presented in the documentation, is the intended curriculum, and that the design of this influences the implemented (or enacted) curriculum in classrooms by teachers. This distinction between different parts of the curriculum is recognised in research (e.g. Nyaumwe, Ngoepe & Phoshoko, 2010) and by education systems in Australia. Education Queensland (2011) for example, recognises five inter-related aspects of curriculum:

- The intended curriculum – what students should learn (documented)
- The enacted (implemented) curriculum – the pedagogy used to shape learning and engage learners
- The experienced curriculum – the curriculum experienced will differ from student to student based on what they bring to the learning context
- The assessed curriculum – the knowledge being assessed and hence prioritised in learning contexts
- The achieved curriculum – what has been learned as a result of what has been taught

The *experienced* curriculum, whilst considered part of the implemented curriculum, is a distinction that requires consideration and attention by teachers, particularly of students with different worldviews. Whilst the intended curriculum is the same for all students, students will experience it in different ways, depending on what they bring with them to the learning situation. Vital (2003) highlighted how teachers may seek to develop a curriculum that will address differences but how that curriculum is enacted in the classroom or school may be quite different from the intended curriculum.

How the intended curriculum is enacted in schools and classrooms is at the behest of the individual teachers. Whilst the Australian Curriculum aims to address an intended curriculum and *influence* the implemented curriculum, this design feature would seem to assume quality teaching in every classroom. The intended curriculum can only become a reality when teachers are able to "deeply understand the goals that learners are expected to achieve, the content learners are to master, and the pedagogies that enable learners to conceptually understand what they learn". (Nyaumwe et al., 2010, p. 65)

Fogarty (2010) points out that, in remote parts of Australia, regardless of the intended curriculum and policy positions, in the end provision "will always come down to a set of formative pedagogic moments where a student either learn or does not". He continues "It is because of this, that the design and nature of the pedagogy that creates and supports these moments must remain a paramount consideration." (p. 217)

In trying to bring some shared sense of quality to the enacted curriculum and the quality of the teaching environment, the National Professional Standards for Teachers (AITSEL, 2011) have been developed. These standards describe the capabilities that teachers need to have and include seven standards, as follows:

1. Knowing students and how they learn
2. Knowing the content and how to teach it
3. Planning for and implementing effective teaching and learning
4. Creating and maintaining supportive and safe learning environments
5. Assessing, and providing feedback and reporting on student learning
6. Engaging in professional learning
7. Engaging professionally with colleagues, parents/caregivers and the community

The link between the intended Australian curriculum and the implemented or enacted curriculum – enabled through these desirable capabilities – is unclear. In particular, 'knowing students and how they learn' requires deep, not superficial knowledge in order to ensure that the experienced curriculum enables the desired achieved curriculum. Moreover, the achieved curriculum should be equitable for all Australians.

Herein is where the development and delivery of a national curriculum may fail to address the needs of all learners. Applying a common learning experience to a group of students may have the appearance of equality – all students have the same learning experience – but they enter the situation with very different backgrounds, including language, culture, knowledge frameworks, life experiences, and so on. These all act as filters to the learning experience that is presented. Ensuring that robust bridges can be built between the knowledges, skills and dispositions of learners that they bring to the formal school context and the intended and enacted curriculum is where issues of equity are most exposed.

Before moving into the body of this chapter, it is important to clarify two theoretical points. First is the notion of equality of access for all students to high-quality schooling. (MCEECDYA, 2008) The notion of a *common curriculum* embeds an approach to social justice that implies an assumption that it is fair and right that all students, schools, families, and/or communities have equal access to common learning. This approach to social justice is underpinned by a belief that inequalities in education, and life more generally, are shaped by inequalities in access to shared or common resources. Such a view, however, fails to consider that not all students enter schools with a common starting point; the provision of a common (intended) curriculum already denies these unequal starting positions.

In contrast, an alternative view of social justice recognises that there are different starting positions for students – some who enter school with different literacy experiences from those of the language of instruction, or from different cultural perspectives where there are differences in orientation to learning or authority; or where there are different aspirations for schooling and life beyond school. Within this view of social justice, greater equity is achieved through providing different learning experiences that try to bridge the differences in starting points.

Having a common (intended) curriculum to which all students have access represents an assumption that this is a fair way of working in a diversely populated country. The provision of the same intended curriculum to all students regardless of location, culture, language or life experience is the conservative mantra that masks the complexities of access to that curriculum as a factor of these variables.

A conservative viewpoint advocates fairness and social justice through equitable provision; all students being treated similarly so that there is no differentiation regardless of background. Knowing that all Australian students will be exposed to a similar curriculum at similar periods of the lifespan rhetorically *seems* to be fair since no child will be at risk of being exposed to an impoverished curriculum. Such a viewpoint suggests that all students and teachers can be expected to have common experiences in terms of their mathematics learning and teaching. Students in inner-city Sydney, the Western suburbs of Melbourne, or the leafy-green affluent suburbs of Perth have the chance to have similar experiences regardless of their wealth, social status, language background, gender or culture. Outwardly, this rhetoric espouses a viewpoint that prioritises a sense of being fair and reasonable to expect that all children will have common experiences across the range of curriculum areas, including mathematics. In adopting this approach, all teachers regardless of their backgrounds will then be able to offer all Australian students access to the same mathematical knowledge regardless of the teachers' or students' background. This standpoint has gained momentum under both Labor and Liberal governments, both of which are conservative in their political and educational orientations.

The rationale for a common curriculum has some roots in the highly differentiated outcomes of national testing practices that have gained increasing precedence in the past decade. A common intended curriculum enables a degree of alignment with the assessed curriculum. This maximises validity which in turn facilitates some degree of reliable reporting of achievement of what is assessed.

The National Assessment Program (NAP) incorporates standardised tests which measure attainment of aspects of the Australian Curriculum by Australian students. The Literacy and Numeracy tests currently measure student *potential* to be literate and numerate, being closely aligned with the National English and Mathematics curricula. Regardless, the assessed curriculum through NAP is not equitable for all Australian students. This type of assessment can privilege select groups of students whilst marginalizing or segregating others (Weinstein, Tomlinson-Clarke & Curran, 2004). This is largely due to the fact that these tests require literacy in the dominant language and consequently are culturally and linguistically biased in spite of the best efforts of producers to ensure otherwise.

Rhodes (1994), working with Indigenous students in the U.S., explains that the genre of most standardized tests requires students to answer quickly, guess, and take risks, skills which many students raised in traditional communities do not have, having been raised to make decisions slowly and accurately. He also notes that in tribal communities (similar to the remote communities of Australia) the norm is to help those in need and work collectively rather than individually. Solano-Flores & Nelson-Barber (2001) similarly argue that 'cultural validity' (i.e. referring to the ways that individuals from different cultures are predisposed to respond to questions and to solve problems) should be a key component in assessment design and implementation. As a consequence, issues of translation, decision-making processes, and rules around teachers offering students support, are three possible factors affecting students' performance on standardized tests.

Further to this, not only do the written questions effectively 'lock out' students from engaging with them on the basis of language used, question contexts can act as a barrier for students who are unfamiliar with them. For example, a numeracy question contextualized in a 'garden centre' about fractional quantities making up 'potting mix' where students have never even heard of either of these concepts (National Assessment Program – Literacy and Numeracy (NAPLAN) – Numeracy Test 2008), neither allows entry nor engagement for second language learners from remote parts of Australia let alone enables them to deal with the mathematics

involved. Such questions are discriminatory and hence do not allow for equitable demonstration of achieved curriculum. Hence the achieved curriculum that is reported on the basis of these tests not only has the potential to be at odds with the intended curriculum, but is more than likely to be markedly different from the implemented and experienced curriculum for the majority of learners from remote schooling contexts. As explained earlier, this is due primarily as a result in inequities in teacher capability in provision, and cultural and language biases in assessment. Currently, the Australian population has been led to believe that data available on the *My School* website (where NAPLAN results for individual scores are available to the public), demonstrates the achieved curriculum. This data is hardly reliable or valid due to the inequities described above.

The National Assessment Program has been the catalyst for considerable reforms and policies in the Australian political and educational landscape. Results from the program, particularly from the literacy and numeracy tests, have shown that there are stark differences in achievement based on the social and economic backgrounds of Australians. The correlation between achievement on NAPLAN and social background is alarming for those in education, although not surprising. From the early 1970s, there has been wide recognition of the differentiated outcomes based on social background internationally (Bowles & Gintis, 1976) and within Australia (Connell, Ashendon, Kessler, & Dowsett, 1982).

To illustrate this correlation, we selected schools with varying 'Index of Community Socio-Educational Advantage' (ICSEA) scores[1] across states of Australia and plotted these scores against the school NAPLAN scores (see Figures 1 & 2). In selecting the schools, we sought to have a comprehensive range of ICSEA scores as the independent variable and then to plot these against NAPLAN outcomes. The graphs reveal the strong correlation between the two variables.

[1] ICSEA indicator is a measure used by ACARA to enable comparisons of schools. It is a school-based measure of relative advantage. It is an amalgam of a range of factors and has been outlined in the fact sheet in the following way:

> ICSEA uses Australian Bureau of Statistics (ABS) and school data to create an index ... The variables that make up ICSEA include socio-economic characteristics of *the* small areas where students live (in this case an ABS census collection district), as well as whether a school is in a regional or remote area, and the proportion of Indigenous students enrolled at the school. (ACARA, 2010)

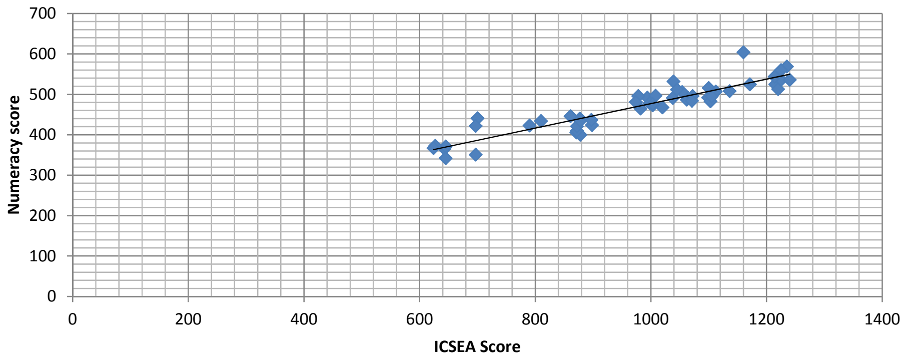

Figure 1. Numeracy result against ICSEA score for Year 5 in some schools, 2009 NAPLAN

From the graph of the Year 5 results, it is clear that there is a strong relationship between the background and location of the students and their scores on the Numeracy test. The higher the social background, as measured by the ICSEA score, the greater the chance that the students (as represented by their school score) will achieve a higher score on the NAPLAN test. The Year 9 results for the same schools shown in Figure 2 below, suggest that these schools have generally been unable to make significant learning gains to the results of students/school over time and indeed, that the relationship between student background and location, and numeracy achievement demonstrated in national tests remains strong over time.

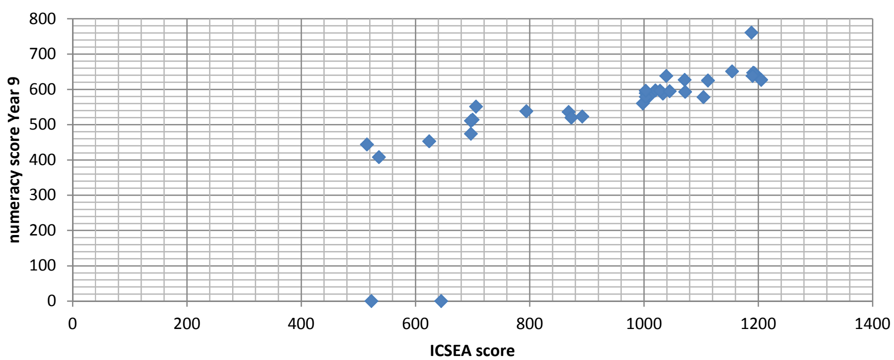

Figure 2. Numeracy result against ICSEA score for Year 9, 2009 NAPLAN

These types of data raise questions for policy makers and educators about the provision of education, particularly for those who are at most risk of failing.

A random selection of almost any remote Indigenous school illustrates the significant differences between performances in literacy and numeracy with that school and other non-Indigenous schools in Australia. In Table 2, the results for Nyangatjatjara College are used to exemplify the disparity in student performance. This College is an Indigenous owned and operated college in Central Australia with 100% of the students being Aboriginal. This College represents many schools in the tri-state region of Australia. This region is the central Australian area that covers South Australian (The APY Lands), the southern part of the Northern Territory and the central eastern part of Western Australian. Unlike the political demarcations of state boundaries that influence education provision, the tristate region is one where the Indigenous families move between states without the usual sense of boundaries. The College is selected merely to represent many of the schools found in this region. While there may be disparities between schools, there are some common trends. According to the *My School* website, 38% of the students have a language background other than English, attendance is 42% and 70 students are enrolled at the College. Costs to operate education are listed at recurrent net income as $27,952 per student. It can be seen from Table 1 that the performance in literacy and numeracy is considerably below that of similar schools on all measures but even worse in comparison with the scores for the national cohort. Such figures highlight the significant differences in performance between many remote schools and mainstream urban settings.

Table 1. Year 7 2010 NAPLAN scores for Nyangatjatjara College (Source: My School Website)

Reading		Writing		Spelling		Grammar and punctuation		Numeracy	
361 (school av)		274 (school av)		404 (school av)		339 (school av)		323 (school av)	
SIM 442	ALL 546	SIM 401	ALL 533	SIM 447	ALL 545	SIM 409	ALL 535	SIM 449	ALL 548

In contrast, there is quite a different story for schools set in urban settings with most of the students at the middle social/economic index. Consider the results for Belmont High School in Victoria (an urban secondary school with of similar size) in Table 2 where a dramatically different story from that of Nyangatjatjara College is shown. Here the ICSEA score is 1029 so it is slightly above the national average social index (of 1000) and has no Indigenous students. Fourteen percent of the students are listed as having a language background other than English. The cost for recurrent expenses for each student is listed as $9,653 so the cost is considerably less than for remote education students. Unlike the remote Aboriginal school, the results at this school are above the national average, albeit only slightly. The data for this school suggest that the demographics are on par with the average socio-economic status of many Australians and that the numeracy scores are comparable with the average scores for Australian students.

Table 2. 2010 year 7 NAPLAN scores for Belmont High School, Victoria (Source: My School Website[2] 2011)

Reading		Writing		Spelling		Grammar and punctuation		Numeracy	
567 (school av)		551 (school av)		554 (school av)		558 (school av)		559 (school av)	
SIM 549	ALL 546	SIM 539	ALL 533	SIM 545	ALL 545	SIM 540	ALL 535	SIM 551	ALL 548

The comparison in achievement of the two schools is stark. While the urban state school is certainly not an elite school in terms of socio-economic index, but rather an 'average' school, it scores considerably higher than the remote Aboriginal secondary college. The comparisons for the economic investment in education are considerable. In the urban setting the costs for education is approximately one third of the cost for remote education provision but there are very large differences in outcomes. The differences in such achievements have been used to justify a national curriculum since one of the concerns regarding remote/indigenous education has been the delivery of an impoverished curriculum that effectively locks students out of progressing through school. While quality of education is a large concern, the economic costs of providing education (quality or inferior) in remote sites are vastly disproportionate to the outcomes that have been achieved. The quality concern has been evident in many of the arguments proposed by Aboriginal educational activists such as Chris Sarra (2007) and Noel Pearson (2009). Providing a quality curriculum to all students is essential for quality and equal learning outcomes. It is without doubt that providing education in remote sites is an expensive venture but extra funding should result in improved educational outcomes for students.

It is widely acknowledged that there are many hard-to-staff schools and that such schools are frequently located in remote locations, in economically-deprived communities, and have high proportions of families from non-mainstream backgrounds (Panizzon & Pegg, 2007). Such schools are often characterised by high turnover of staff, inexperienced staff, or staff who are not trained in the areas in

[2] 'SIM' = average for statistically similar schools; 'ALL' = average for all Australian schools; Factors used to determine a group of similar schools are the socio-educational backgrounds of the students' parents, whether the school is remote, the proportion of Indigenous students, the proportion of students from a language background other than English, or a combination of these factors

which they teach. Collectively, these factors are likely to contribute to the capacity of staff to implement/enact the curriculum as it is designed.

Whilst quality and consistency is enabled through the Australian Curriculum, the equity in student achievement still resides in variations of teacher quality. As previously indicated, the implemented curriculum depends primarily on the quality of teachers. This quality is, in turn, affected by resources and facilities (some of which depend on location), and the knowledge and skill base of teachers, which can also depend on location since access to professional learning and development is critical in developing teacher capability.

Geography and Education Provision

The stark differences between states and territories require an askance look as to how these very different states are able to offer similar educational opportunities for their constituents. Table 3 below enables a comparison between the population and area of each state/territory. The population of the entire Northern Territory is comparable with some major cities in other states (such as Geelong in Victoria or Wollongong in New South Wales) yet the Northern Territory is the third jurisdiction in the nation by area. The challenge in providing education to so few people over such a large area is enormous. The implications for policy rollout or teacher professional development are considerable in this context. The Northern Territory has previously assumed responsibility for developing its own curriculum and supporting resources (other than the senior curriculum which is shares with South Australia). However, with such a small population, the responsibility for such development has relied on a small body of people. Logic suggests that in comparing population alone, the multiplier effect between the Northern Territory and states such as Victoria (25 times greater) or New South Wales (33 times greater) means that there are many more people available and greater expertise to draw from, to work on curriculum support resources in these states than in the Northern Territory.

The small population, vast distances, the fact that most community schools are small in terms of the numbers of teachers, and that most Aboriginal and Torres Strait Islander people are living in remote communities, creates a unique set of circumstances in The Territory. Providing professional development for teachers is challenging not only in terms of delivery where distances are vast but even satellite technologies are not so reliable for on-line delivery. Many of the teachers are early career teachers, and the staff turnover in remote schools is high. (Roberts, 2004) Implementing and sustaining reforms, such as the national curriculum, become challenging under these circumstances. In such a context the need for, and capacity to deliver, professional development for teachers in remote areas is quite different from urban, centralised states and systems.

Table 3. Populations and Areas for Australian States and Territories[3]

	Area (sq kms)	Estimated resident population (millions)	Capital	Estimated resident population (millions)
Australia	7 692 024	21.01		
New South Wales	800 642	6.89	Sydney	4.34
Victoria	227 416	5.21	Melbourne	3.81
Queensland	1 730 648	4.18	Brisbane	1.86
Western Australia	2 529 875	2.11	Perth	1.56
South Australia	983 482	1.58	Adelaide	1.16
Tasmania	68 401	0.49	Hobart	0.21
Australian Capital Territory	2 358	0.34	Canberra	0.34
Northern Territory	1 349 129	0.21	Darwin	0.12

While geography and population are considerable in their shaping and constraining of possibilities, a further consideration is the diversity within regions. Culture and language must be considered as these vary considerably across the nation. For example Victoria, New South Wales and Tasmania are quite different in their makeup of Indigenous people. While the majority of Indigenous Australians reside in these states, they are very different from the Indigenous people who live in remote parts of Northern Territory, Queensland, Western Australia and South Australia.

In terms of cultural differences, the contrasts between the states and territories are also quite marked. In the Northern Territory, approximately 30% of the population is Indigenous which is in stark contrast from the national average of 2%. Almost 50% of school-age students in the Northern Territory are Indigenous. Most of these people live in remote communities in contrast to many of the more populous states where almost all the Indigenous Australians live in urban or regional centres. There are approximately 30 Indigenous languages in the Northern Territory with the remote students often learning and speaking English as a foreign language. For many remote students living in community, their only encounter with English is in the school classroom. With traditional cultural practices still very strong in many remote parts of the country, there are further challenges to the orthodoxies embedded in curriculum as to whose worldview is represented in and through the curriculum.

[3] Source: Department of Foreign Affairs and Trade (2011) using ABS 2007 census data

Teachers working in these contexts require a strong degree of cultural competence. Villegas and Lucas (2002) use six characteristics to describe culturally responsive teachers: They

1. are socio-culturally conscious
2. have affirming views of students from diverse backgrounds
3. see themselves as responsible for and capable of bringing about change to make schools more equitable
4. understand how learners construct knowledge and are capable of promoting knowledge construction
5. know about the lives of their students, and
6. design instruction that builds on what their students already know while stretching them beyond the familiar.

It can be seen that all of these characteristics require knowledge of the students that teachers are working with, and in particular, the communities and worldviews that they bring with them to the learning environment.

Worldviews and Dominant Forms of Knowledge

As educators, it is critical to appreciate the very different worldviews of Aboriginal people. Trying to represent complexities of cultures and worldviews in and through curriculum documents is a difficult challenge. In the current National Curriculum mathematics document, the framework is very heavily located in Western worldviews and represents the dominant Anglo centric worldview. As such, it excludes Aboriginal and other non-dominant cultures (including, for example, working-class, Muslim or non-Anglo cultures). Some attempts are made to embed examples in non-dominant forms as if such tokenistic examples may be seen to be inclusive of different cultures. The framework itself preserves the status quo that the mathematics curriculum is a set of objective and reified facts that are transcultural (Bishop, 1988).

However, culture is much more omnipresent than simple examples. By way of example, at one point during our time in Central Australia, a large storm hit the area with considerable rain falling. The following day, at a meeting, a comment was passed by a non-Aboriginal person about the intensity of the storm to which one of the Elders commented “thank you” as if the comments were being passed as a compliment about the rain storm. This confused many of the non-Aboriginal people. Further conversations followed in which it was explained that on the way home that evening the Elders had seen the rain clouds and ‘sang’ the rain. They felt that the rain had been sung into existence as a consequence of their actions. They projected a certain pride in their success in singing the rain at a time when it was needed. Sometime later, when no rain had fallen, a non-Aboriginal person asked one of the Elders if they could sing some more rain. The response was something like "We can't sing any rain, there are no clouds" but the facial expression of the Elder was one of incredulous disbelief as if the requestor was not very intelligent by not recognizing the impossibility of the task. This story struck a strong cord with us as it illustrated the very different ways of seeing and living in the world. In this case, the worldview of how rain came into existence was elusive for us but quite apparent for the Elder. In such a context, it becomes quite a challenge to teach the national science curriculum where the water cycle is explained in a very different way. These contradictory worldviews make for very different learnings for both in-school and out-of-school that may be quite challenging for remote Indigenous students to reconcile. While this has clear implications for the implementation of the science curriculum, how the difference resonates with the mathematics curriculum is somewhat more salient.

Similar differences in the mathematical worlds can be observed. In the Western world, mathematics is often seen as a body of knowledge that represents pre-existing facts and, hence is irrefutable (Ernest, 1991). Being complicit with this worldview results in the mathematical curriculum being taken for granted and being seen to represent a body of facts that transcend cultures, languages and societies. However, much like the rain example, mathematics is equally a representation of the culture/s from which it emanates or within which it exists. For example, in Western cultures, the capitalist imperative shapes the desire to measure and quantify how the world is seen and described. This is starkly contrasted with desert people for whom the world is much more focused on immediate issues, such as food and water. Thus the desire to count is superseded by a very different imperative which may, for example, focus on how edible a food source may be rather than how much (many) of that food source is available. Thus, what remains a greater challenge for curriculum development and implementation is to identify the ways in which the mathematics is shaped by the culture and then to try to build bridges between the two knowledge forms.

In the central desert context, the Indigenous people speak their home languages which are shaped by, and also shape, their worldviews. In Pitjantjatjara, for example, the language is quite restricted in terms of number concepts. The lands of the desert are quite stark with few resources so the need for a complex language for number is limited. As such, the counting system is one of 'one, two, three, big mob'. It is rare that a collection of three or more occurs so the need for a more developed number system is not apparent. Even when living in community, the need for number is limited. Few people are aware of their birthdates, and numbers in community are very limited in terms of home numbers or prices in the local store. As such, the immersion in number that is common in urban and regional centres is

very limited in remote communities. Therefore, many of the taken for granted assumptions about number that are part of a standard curriculum are limited in this context. This makes teaching many mathematical/number concepts quite challenging as it is not only the teaching of mathematical concepts and processes but a process of induction into a new culture and new worldview. In many communities there is a strong resistance to learning many of the concepts in the curriculum with the frequently asked question of "Why do we have to learn this?" as the knowledge is not relevant to their daily lives. Similarly there is a strong resistance in many communities for instruction in Standard Australian English. In the past year, the Northern Territory government dismantled its formal bi-lingual education program but in other contexts, such as Fitzroy River in the Kimberley, there has been a strong push for the first few years of schooling to be in the home language while the students transition into school English.

Pitjantjatjara is also a very gestural language so that the people may use considerable body language and intonations to communicate. In asking directions to a location, the body will be tilted in the direction of the site – this may be with a hand or head movement. The distance is unlikely to be articulated as a form of measure but the intonation of the wording will give a sense of distance. The speaker is likely to say – "him long way" or "him looooong way" to differentiate the distance to be travelled. Similarly, comparative terms such as long, longer or longest are not found in traditional languages so the intonation placed on words aids in differentiating and comparisons. Whilst these are forms of measurement, they are neither acknowledged nor valued in the national curriculum.

Similarly, the prepositions found in Pitjantjatjara are limited to approximately six in comparison with approximately 64 in Standard Australian English. Where these differences in language are so extreme, coming to learn many concepts, mathematical or otherwise, is challenging as students' home language fails to resonate with the language of instruction and hence many key ideas are unable to be grasped due to the differences in the structures of language.

Many of the local languages and dialects are shaped by the environments but also shape the ways of seeing and acting in the social world. When the home language does not have the language structures that are an integral part of the school curriculum, it becomes an imperative for teachers to build ways of understanding and respecting home languages in order to support the transition from home languages to the school/mathematics language.

Balance Between Common Curriculum and Life-skills

When the differences between Indigenous and non-Indigenous worldviews, epistemologies, and knowledge frameworks form such a chasm, questions about the appropriateness of a National Curriculum should be asked. In this case, what can be observed is that remote Indigenous students are performing significantly below the national average. However, scores belie the real issues – social, economic, as well as educational. In this scenario, remote Indigenous students are most at risk of leaving school with very low levels of literacy and numeracy, or worse still, being illiterate and innumerate. During our time in Central Australia, an employer approached one of us with regard to a graduate of the school. This student, who was now in his mid-twenties, had applied for work but the only sight word he recognised was his name. While this may be an extreme example, it highlights the difficulties school graduates have when seeking employment and/or on being placed in a work situation. Being illiterate and innumerate severely limits the possibilities of employment and employability. Such outcomes can be seen as a product of the complex mixture of curriculum, pedagogy, and attendance which in turn is a complex mixture of social and cultural factors.

A national curriculum may offer consistency across the nation through affordability of, and hence access to, curriculum resource materials. It also has the potential to ensure that all Australians are exposed to a rich and deep curriculum, particularly those students who traditionally have been offered a restricted curriculum based on their social, cultural or geographic location. It might also be used to ensure, via some accountability measures, that teachers are offering the best curriculum to their students. As many Aboriginal education activists are espousing, too many Aboriginal students have been exposed to a deficit curriculum and pedagogy that has effectively locked them out of education and, ultimately, work. But is this sufficient for justifying a common curriculum? The goals of the nationally documented curriculum may fail due to the inability to reconcile the intended curriculum with the enacted and experienced curriculum.

While the Australian Curriculum may help in some ways to offer a standard, high-stakes curriculum to all Australians, and most notably, to those most at risk of poor education provision and outcomes, it is a framework for knowledge production. Much still rests with the individual teachers. The Australian Curriculum in mathematics does not offer a 'way' to teach school mathematics. Herein, is a very significant variable in terms of quality education provision that has not been explored in the document. The pedagogical framing of the knowledge structures outlined in the document may offer hope, or doom, for learners. While teachers may have the content prescribed in terms of 'what' they teach, there is still considerable scope for 'how' that knowledge is taught. The experienced curriculum becomes a much more substantial issue in terms of equity and outcomes for the students.

Fogarty (2010) describes 'place based pedagogies' that enable a greater connection between the lived experience and aspirations of Indigenous students and their communities, and schooling and work. He suggests that a "pedagogical framework is needed to enable the inclusion of Indigenous knowledge in pedagogic design and a connection between this knowledge and Indigenous development realities in remote communities" (p.218).

For example, addition of fractions may be taught as rote manipulation of numbers in one context, or using concrete resources to represent the addition process by another. Still others may use 'place based pedagogies' and draw substantially on community activities to provide examples of where and how fractions are used in everyday lives so that the knowledge can be made relevant and meaningful. In so doing, that teacher may create very different opportunities for learning the same content knowledge. Teachers will have considerable scope in how they might bring about equity in their classrooms. It may not be in terms of intended learning as indicated in the Australian Curriculum but through their pedagogical processes, where cultural, linguistic or geographical diversity can be built into the lives of mathematics classrooms.

In this chapter, we have sought to raise a number of issues. First is that too many remote Indigenous students leave school functionally illiterate and innumerate at a cost that is economically disproportionate with those of their urban peers. Assuming students continue to attend school regularly in order to build and maintain mathematical knowledge structures and processes as they progress through school, their capacity to engage with many mathematical concepts and processes is hindered on two key fronts. First is the literacy of mathematics, which includes the language of mathematics, and second is the ways in which the pedagogical processes including teacher-student interactions are undertaken in the classrooms. Many of these practices are dissimilar to those of the home and hence lock students out of participating effectively in the classroom. Coming to learn mathematics is as much about cultural induction as it is about the mathematics per se. In cultures where the gap between the home culture and that of the school is not significant, then the work of the teacher is somewhat easier than where the cultural gap is a chasm as is in the case of many remote Indigenous communities.

It is at this point the question of how a national curriculum may serve remote Indigenous students must be asked. While this group of students are the most likely to fail on national measures of achievement, they represent one end of the educational continuum. Other social, cultural and linguistic groups located in various sites around the nation experience some or more of the issues addressed in this chapter, albeit to differing extents. In these circumstances, it is important to consider how a national curriculum can meet the needs and backgrounds of the diverse groups in this country. Having a curriculum that describes the intended learning for all students is one thing. Ensuring that the intended curriculum is enacted and experienced by students in ways that, despite their variation, ensure learning is maximised for each and every student, is another. Equity in provision does not guarantee equitable learning outcomes. The quality of the enacted curriculum is most likely to bring about improvements in learning outcomes; teachers, schools and systems must address this aspect of curriculum if there are to be changes to the possibilities of learning mathematics for all Australian students.

If the purpose of formal schooling is to prepare citizens for the world beyond schools in terms of knowledge production and dispositions for mainstream society, then all students must have access to the knowledge systems that empower them and enable them to make successful transitions into the world. A robust national curriculum is essential to enable and promote high expectations and equity in access. And whilst the National Professional Standards for teachers offers an attempt to ensure equity in teacher quality, governments must do more to address disadvantages for teachers created through geographical location and distance. Quality teachers who are able to develop appropriate practices that will enable students to enter into the world of school mathematics successfully are needed in remote sites. This is the next step in the process of enabling all Australian students success in school mathematics regardless of their language, cultural background, gender or geographical location.

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Chapter 7
Digital Technologies in the Australian Curriculum: Mathematics – A Lost Opportunity?

Merrilyn Goos

The initial framing documents that informed development of the Australian Curriculum – Mathematics noted that digital technologies could offer new ways of teaching and learning mathematics that help deepen students' understanding. This chapter examines the extent to which this aim has been achieved. It uses two research-based frameworks to analyse the technology messages in the publicly available versions of the Foundation to Year 10 and senior secondary mathematics curricula. The first framework uses the metaphors of *master*, *servant*, *partner*, and *extension of self* to gauge the extent to which technology transforms teaching and learning roles. The second classifies pedagogical opportunities afforded by technology in terms of changes to *tasks*, *classroom interactions*, and the *subject* of mathematics itself. In all curricula, expected uses of technology were found to be mostly consistent with the *servant* metaphor in that technology was referred to as a more efficient method for calculating or graphing. Pedagogical opportunities afforded by the curriculum were typically restricted to the level of *tasks*, where technology could be used to link representations or work with real data. Thus in this new curriculum for Australian schools, the potential for technology to support new classroom practices and curriculum goals remains unfulfilled.

Digital technologies have been available in school mathematics classrooms since the introduction of simple four function calculators in the 1970s. Since then, computers equipped with increasingly sophisticated software, graphics calculators that have evolved into "all-purpose" hand held devices integrating graphical, symbolic manipulation, statistical and dynamic geometry packages, and web-based applications offering virtual learning environments have promised to change the mathematics teaching and learning landscape. But what should be the role of digital technologies in school mathematics? Is technology meant to help students get "the answer" more quickly and accurately, or to improve the way they learn

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mathematics? It is particularly timely to consider this question when the new *Australian Curriculum: Mathematics* is about to be implemented in schools.

This chapter considers relationships between technology-related research and teaching practice and the development of the *Australian Curriculum: Mathematics*, and implications for supporting effective mathematics teaching and learning in Australian schools. The first part of the chapter considers key messages from research on learning and teaching mathematics with digital technologies. The second part offers some snapshots of practice to illustrate what effective classroom practice can look like when technologies are used in creative ways to enrich students' mathematics learning. The third part analyses the technology messages contained in version 1.2 of the *Australian Curriculum: Mathematics* (ACARA, 2011) and the challenges of aligning curriculum policy with research and practice.

Key Messages from Research on Learning and Teaching Mathematics with Digital Technologies

Fears are sometimes expressed that the use of technology, especially handheld calculators, will have a negative effect on students' mathematics achievement. However, meta-analyses of published research studies have consistently found that calculator use, compared with non-calculator use, has either positive or, at worst, neutral effects on students' operational, computational, conceptual and problem solving abilities (Ellington, 2003; Hembree & Dessart, 1986; Penglase & Arnold, 1996). In addition, it is often found that students who used calculators during class report more positive attitudes towards mathematics than their counterparts who did not use calculators. While rigorously conducted meta-analyses reassure us that calculator use does not erode students' mathematical skills or conceptual understanding, we may question some of the assumptions on which such analyses are based. In particular, meta-analyses typically select studies that compare treatment (calculator) and control (non-calculator) groups of students, with the assumption that the two groups experience otherwise identical learning conditions. They inquire into the *effects* of calculator use on students' mathematical achievement. However, experimental designs such as this do not take into account the possibility that technology fundamentally changes students' mathematical practices and even the nature of the mathematical knowledge they learn at school. A different research question that needs to be addressed is: What *changes* when students and teachers use digital technologies for learning mathematics?

Technology and Mathematical Knowledge

In their contribution to the *17th ICMI Study on Mathematics Education and Technology*, Olive and Makar (2010) analysed the influence of technology on the nature of mathematical knowledge as experienced by school students. They argued as follows:

> If one considers mathematics to be a fixed body of knowledge to be learned, then the role of technology in this process would be primarily that of an efficiency tool, i.e. helping the learner to do the mathematics more efficiently. However, if we consider the technological tools as providing access to new understandings of relations, processes, and purposes, then the role of technology relates to a conceptual construction kit. (p. 138)

Their words encapsulate the contrasting purposes of technology that were foreshadowed in the opening paragraph of this chapter. For learners, mathematical knowledge is not fixed but fluid, constantly being created as they interact with ideas, people, and their environment. When technology is part of this environment, it becomes more than a substitute or supplement for mathematical work done with pencil and paper. Consider, for example, the way in which dynamic geometry software allows students to transform a geometric object by "dragging" any of its constituent parts and thereby to investigate its invariant properties. Through this experimental approach, students make predictions and test conjectures in the process of generating mathematical knowledge that is new for them. Technology can change the nature of mathematical knowledge and the environments in which students learn mathematics. This potential for change should encourage us to reconsider what counts as foundational knowledge to be included in the school mathematics curriculum.

Technology and Mathematical Practices

Learning mathematics is as much about *doing* as it is about *knowing*. How knowing and doing come together is evident in the mathematical practices of the classroom. For example, school mathematical practices that, in the past, were restricted to memorising and reproducing learned procedures can be contrasted with mathematical practices endorsed by most modern curriculum documents, such as conjecturing, justifying, and generalising. Technology can change the nature of school mathematics by engaging students in more active mathematical practices such as experimenting, investigating, and problem solving that bring depth to their learning and encourage them to ask questions rather than only looking for answers (Farrell, 1996; Makar & Confrey, 2006).

Olive and Makar (2010) argue that mathematical knowledge and mathematical practices are inextricably linked, and that this connection can be strengthened by the use of technologies. They developed an adaptation of Steinbring's (2005) "didactic triangle" that in its original form represents the learning ecology as interactions between student, teacher, and mathematical knowledge (Figure 1). Introducing technology into this system transforms the learning ecology so that the triangle becomes a tetrahedron, with the four vertices of student, teacher, task and technology creating "a space within which new mathematical knowledge and practices may emerge" (p. 168). (See also Sträßer, 2009, for a related discussion of the relationship between teacher, student, mathematical knowledge, and artefacts such as digital technologies.)

Within this space, researchers have theorised new relationships between technology and its users. Arguing from a sociocultural perspective, Goos, Galbraith, Renshaw, and Geiger (2000) describe digital technologies as cultural tools that not only amplify, but also re-organise, cognitive processes through their integration into the social and discursive practices of a knowledge community. Learning is amplified in a quantitative sense when technology is used to speed up tedious calculations or to verify results obtained first by hand. A more profound cognitive re-organisation occurs when students' thinking is qualitatively transformed through interaction with technology as a new system for meaning-making. Other researchers take the position that not only does technology shape the knowledge constructed by students, but the tool itself is also transformed in a process described as instrumental genesis. Drawing on the instrumental approach developed by Vérillon and Rabardel (1995), Artigue (2002) explains that through this process a material or symbolic object, or "artefact", becomes an "instrument" through construction of personal schemes of use. Digital technologies are thus artefacts that have no instrumental value until they are used, and eventually transformed, for carrying out specific tasks. They become instruments when the individual constructs schemes of use or appropriates social pre-existing schemes.

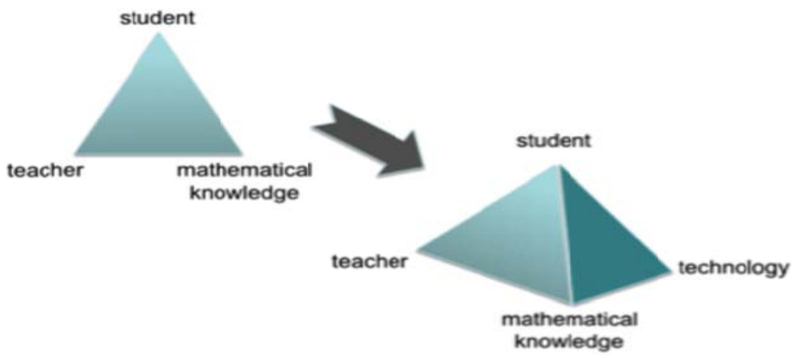

Figure 1. The didactic triangle becomes the didactic tetrahedron

Frameworks for Analysing Mathematical Practices

Within the tetrahedral space imagined by Olive and Makar (2010), students and teachers may imagine their relationship with technologies in different ways. Goos, Galbraith, Renshaw and Geiger (2000) developed four metaphors to describe how technologies can transform teaching and learning roles. Technology can be a *master* if students' and teachers' knowledge and competence are limited to a narrow range of operations. Students may become dependent on the technology if they are unable to evaluate the accuracy of the output it generates. Technology is a *servant* if used by students or teachers only as a fast, reliable replacement for pen and paper calculations without changing the nature of classroom activities. Technology is a *partner* when it provides access to new kinds of tasks or new ways of approaching existing tasks to develop understanding, explore different perspectives, or mediate mathematical discussion. Technology becomes an *extension of self* when seamlessly integrated into the individual and collective practices of the mathematics classroom.

Pierce and Stacey (2010) offer an alternative representation of the ways in which technology can transform teachers' mathematical practices. Their *pedagogical map* classifies ten types of pedagogical opportunities afforded by a wide range of mathematical analysis software. Opportunities arise at three levels that represent the teacher's thinking about:

- the *tasks* they will set their students (using technology to improve speed, accuracy, access to a variety of mathematical representations, or for working with real data or simulated real life situations);

- *classroom interactions* (using technology to change the classroom social dynamics or to change the didactic contract that governs students' and teachers' expectations of each other's roles);
- the *subject* being taught (using technology to provoke mathematical thinking, support new curriculum goals, or change the sequencing and treatment of mathematical topics).

Snapshots of Classroom Mathematical Practice

Three vignettes are presented here to illustrate how technology can be used creatively to support new mathematical practices. Each is analysed with reference to the two frameworks outlined in the previous section.

Vignette #1: Representing Irrational Numbers

Geiger (2009) used the master-servant-partner-extension-of-self framework to analyse a classroom episode in which he asked his Year 11 students to use the dynamic geometry facility on their CAS calculators to draw a line $\sqrt{45}$ units long. His aim was to encourage students to think about the geometric representation of irrational numbers. The anticipated solution involved using the Pythagorean relationship $6^2 + 3^2 = \left(\sqrt{45}\right)^2$ to construct a right angled triangle with sides 6 and 3 units long and hypotenuse $\sqrt{45}$ units long (shown in Figure 2).

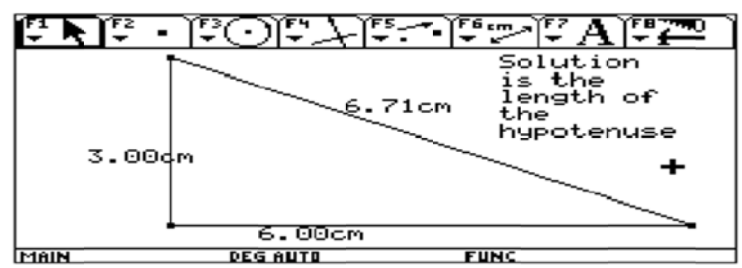

Figure 2. Drawing a line $\sqrt{45}$ units long

Students began by using their calculators in *servant* mode to find the square root of various numbers. They passed the calculators back and forth to share and critique each other's thinking. Because the calculator was used here as a medium to make thinking public, it took the role of a *partner*. The teacher invited one student to present her results to the class – the square root of 45 expressed to ten decimal places, which she assumed was a terminating decimal because the calculator was capable of displaying up to 12 decimal places. Other students pointed out that $\sqrt{45}$ is irrational, however, and cannot terminate. They helped the presenting student change a setting on the calculator that restricted the number of decimal places displayed. For some students, then, technology acted as their *master* because of their lack of familiarity with how to control the display features of the CAS calculator.

The teacher decided to give the class a hint, by making public a comment made by one student relating to triangles.

Teacher: But what would it *look* like?
Sam: Well, you could have a triangle…
Teacher: (to class) I think Sam's given you a hint!
Nicole: Has it got something to do with Pythagoras?
Teacher: Way to go!

Students began to use their calculators in *servant* mode once more, searching for Pythagorean triples without relating this to the geometric representation as instructed by the teacher.

Susie: So we're trying to relate it to 45! The hypotenuse.
Nicole: So the side of "a" could be √5 and the side of "b" could be √40, so √40 squared and √5 squared is 45.
Susie: But the length of the sides will be an irrational number.

Before long, the teacher redirected the class to consider geometry, not just numbers.

Teacher: There seems to be a lot positively related to the work we were doing yesterday, but walking around, there were five of you doing geometry and the rest of you were on your calculators working only with numbers. So, some of us are going to have to take a little risk and get out of our comfort zones. We like working with numbers because it's comfortable, but just because you're busy doesn't mean it's productive. Other people have given you big hints. You need to try to work with that.

Despite this advice, most students continued to search for a numeric solution, that is, by using their calculators merely as a *servant*. Nevertheless, one student, who had previously been working alone, joined the discussion in his group to convince the others that his solution was appropriate. He passed his calculator around the group, in *partner* fashion, to support his investigation of the task.

Diane: We've got two that are the same, so 45 = 9 + 36.
Gena: Yeah, so that equals 9. So write that down!
Karen: Tom wants your attention.
Gena: What are you trying to say Tom?

Tom: No. Because that squared plus that squared … whatever the number what … 6 point something … so ... To get that number, you need a right-angled triangle with a side of 6 and 3 and that -
Frances: I get it now!
Harry: How do you know it's going to be a rational number?
Tom: That! That's the root of 45. If you want to know how to draw it -
Harry: That line is going to be …
Tom: By drawing that and that and the right angle, you get that.
Gena: Yay! Cool!

In this episode, technology was initially used as a *servant* to perform numerical calculations that did not lead to the desired geometric solution. It became a *partner* when students passed their calculators around the group or displayed their work to the whole class to offer ideas for comment and critique. As a *partner* it gave the student who found the solution the confidence he needed to introduce his conjectured solution into a heated small group debate. In terms of Pierce and Stacey's (2010) pedagogical map, this episode illustrates opportunities provided by a *task* that links numerical and geometric representations to support *classroom interactions* where students share and discuss their thinking.

Vignette #2: Creating an Exponential Model

Geiger, Faragher, and Goos (2010) investigated how CAS technologies support students' learning and social interactions when they are engaged in mathematical modelling tasks. In this snapshot, Year 12 students worked on the following question:

When will a population of 50,000 bacteria become extinct if the decay rate is 4% per day?

One pair of students developed an initial exponential model for the population y at any time x, $y = 50000 \times (0.96)^x$. They then equated the model to zero in order to represent the point at which the bacteria would be extinct, with the intention of using CAS to solve this equation. When they entered the equation into their CAS calculator, however, it unexpectedly responded with a *false* message, shown in Figure 3. The students thought this response was a result of a mistake with the syntax of their command. When they asked their teacher for help, he confirmed their syntax was correct but said they should think harder about their assumptions.

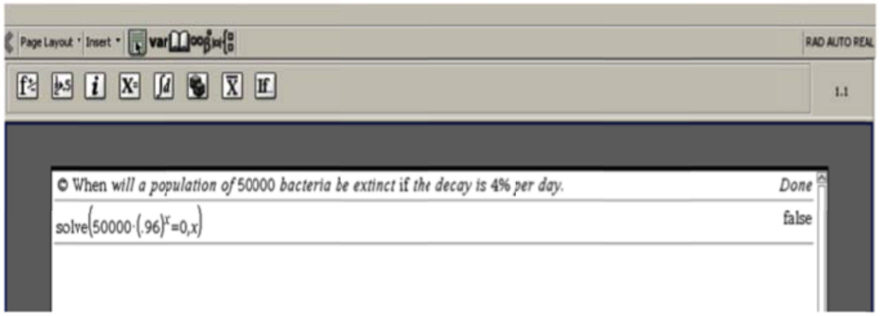

Figure 3. Calculator display for the problem $y = 50000 \times (0.96)^x$

Eventually, the teacher directed the problem to the whole class and one student spotted the problem: "You can't have an exponential equal to zero". This resulted in a whole class discussion of the assumption that extinction meant a population of zero, which they decided was inappropriate. The class then agreed on the position that extinction was "any number less than one". Students used CAS to solve this new equation and obtain a solution.

In this episode the teacher exploited the "confrontation" created by the CAS output to promote productive interaction among the class (technology as *partner*). Using this pedagogical opportunity also allowed the teacher to capitalise on the way the calculator displayed an error message to provoke mathematical thinking, thus fulfilling *subject* level goals of promoting thinking about the mathematical modelling process rather than practising skills.

> Teacher: It was pretty obvious to me why it didn't work but I deliberately made a point of that with a student to see what their reaction would be. And it was a case of pretty much what I expected. That they just grasped this new technology *Nspire* and were so wrapped up in it that they believed it could do everything and they didn't have to think too much. And so suddenly, when it didn't work, it took a fair amount of prompting to get them to actually go back and think about the mathematics that they were trying to do and why it did not give a result.

Interestingly, the teacher noted that even if the students were solving the equation without technology, they would still need to have this discussion about the meaning of "extinction". However, he used the error display to challenge their assumption that the calculator was a "black box" that would always produce the correct answer.

Vignette #3: Analysing Personal Data

This final example is drawn from a study that helped teachers plan and implement numeracy strategies across the middle years curriculum (Geiger, Dole, & Goos,

2011; Goos, Geiger, & Dole, 2011). Teachers were introduced to a model of numeracy whose elements comprise mathematical knowledge, dispositions, tools, contexts, and a critical orientation to the use of mathematics. The incorporation of tools into the model emphasised the ways in which symbolic tools and other artefacts "enable, mediate, and shape mathematical thinking" (Sfard & McClain, 2002, p. 154). In mathematics and non-mathematics classrooms, tools may be representational (symbol systems, graphs, maps, diagrams, drawings, tables), physical (models, measuring instruments), and digital (computers, software, calculators, internet).

As part of the study, one teacher developed an activity within her Year 8 Physical Education program where students investigated their level of physical activity through the use of a pedometer that they wore for one week. In previous years when completing this task, students had recorded their personal data in a table in their notebooks. The teacher would demonstrate the procedure for converting the number of steps to kilometres and instruct students to draw a bar graph to show how many km per day they had walked. However, as a result of her participation in the study, the teacher was now experimenting with a different, less directive approach that would encourage students to take a more active role in their learning.

Every day, students entered their data – the number of paces they had walked or run – into a shared Excel spreadsheet. They analysed their own data by using facilities within Excel, for example, the graphing tool, and then compared their results with those of other students (see Figure 4).

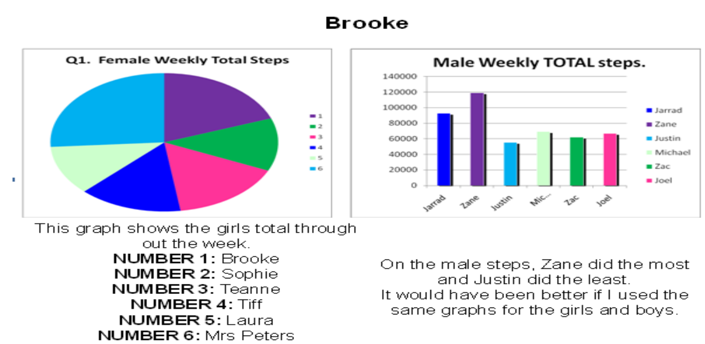

Figure 4. A comparison of males' and females' weekly total steps.

As part of this analysis, students were asked to convert their total daily and total weekly paces into kilometres to gain a sense of how far they typically walked in the course of a day or a week. The task was also designed to help students realize that the distance they walked was not determined by the number of paces alone as an individual's pace length was also a factor. This was the first time that the teacher had introduced digital technologies into her practice, so she relied on students in the class who were familiar with spreadsheets to help their peers, and also to help her. Students entered the daily numbers of steps recorded on their personal pedometers into a class spreadsheet that the teacher displayed via a newly installed interactive whiteboard. They used Excel formulas to calculate the total number of steps per week and the number of kilometres this represented for each student in the class. Students chose charts from the Excel menu to display their data and make comparisons, for example, between steps taken on different days of the week for one student, or between total steps taken by a male and a female student (Figure 4). If students chose inappropriate graphs for these purposes (e.g., line graphs), the teacher questioned them to draw out the reasons and lead them to identify better ways of representing their data.

The teacher and students used the spreadsheet as a *servant* to record, analyse and represent data; but this tool, together with the interactive whiteboard, was also a *partner* that mediated discussion between students in relation to differences they observed as they critically compared their own results to those of others and attempted to explain the differences. From the teachers' point of view, introducing the spreadsheet helped achieve her goal of changing *classroom interactions* by giving students a greater sense of authority and positioning the teacher as a co-investigator.

Aligning Curriculum with Research and Practice? Technology in the *Australian Curriculum: Mathematics*

The brief research summary and classroom snapshots presented above show how digital technologies provide a "conceptual construction kit" (Olive & Makar, 2010, p. 138) that can transform students' mathematical knowledge and practices. This is not a new claim; nor has the transformative potential of digital technologies been ignored in previously published curriculum documents and educational policy recommendations. For example, the National Council of Teachers of Mathematics (2000) lists six guiding principles for the content and character of school mathematics. These principles concern equity, curriculum, teaching, learning, assessment, and technology. The NCTM's Technology Principle states that "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24). At around the

same time as the NCTM's *Principles and Standards for School Mathematics* were published, the Australian Association of Mathematics Teachers (2000) issued a communiqué on graphics calculators and school mathematics, which said: "There is a compelling case for the advantages offered to students who use graphics calculators when learning mathematics. They are empowering learning tools, and their effective use in Australia's classrooms is to be highly recommended". To what extent does the *Australian Curriculum: Mathematics* support this view of technology?

Early Versions of the Curriculum

The shape paper that provided the initial outline of the mathematics curriculum (National Curriculum Board, 2009) made it clear that technologies should be embedded in the curriculum "so that they are not seen as optional tools" (p. 12). Digital technologies were seen as offering new ways to learn and teach mathematics that helped deepen students' mathematical understanding. It was also acknowledged that students should learn to choose intelligently between technology, mental, and pencil and paper methods.

The draft consultation version 1.0 of the Foundation to Year 10 (F-10) mathematics curriculum expected "that mathematics classrooms will make use of all available ICT in teaching and learning situations". The intention was that use of ICT was to be referred to in content descriptions and achievement standards. Yet this was done superficially throughout the first published version of the curriculum, with technology often being treated as an add-on that replicates by-hand methods. Technology was mentioned in only 27 (14.6%) of the 185 content statements in this version of the curriculum, and 17 of these statements referred to technology in the following terms:

> Plot graphs of linear functions and use these to find solutions of equations *including use of ICT* (Year 8 Number and Algebra strand, emphasis added).
>
> Construct, read, interpret and make connections between tables and simple graphs with many-to-one correspondence between data and symbols, *including using ICT* (Year 4 Statistics and Probability strand, emphasis added).
>
> Visualise, demonstrate and describe the effects of translations, reflections, and rotations of two-dimensional shapes and describe line and simple rotational symmetry, *including use of ICT* (Year 5 Measurement and Geometry strand, emphasis added).

The implication of statements such as this is that ICT methods should do no more than provide an alternative, and perhaps quicker, method of completing the task, rather than transforming the nature of the task itself in developing students' mathematical understanding.

In the corresponding consultation versions of the four senior secondary mathematics courses, the aims for all courses referred to students choosing and using a range of technologies. Nevertheless, each course contained a common technology statement – "Technology can aid in developing skills and allay the tedium of repeated calculations" – that suggested a limited view of its role as nothing more than an "efficiency tool" (Olive & Makar, 2010, p. 138) that supplements pencil and paper calculations. There is no evidence here that the curriculum is informed by research that shows how technology can open the gateway to deeper understanding and exploration of mathematical concepts. Across the courses, variable messages about the use of technology were conveyed in words like "assumed" and "vital" in Essential and General Mathematics, to "should be widely used in this topic", "can be used to illustrate practically every aspect of this topic", or no mention at all for some topics in Mathematical Methods and Specialist Mathematics. An unfortunate implication of this variable treatment was that technology is only considered to be valuable for less able students and only for getting "the answer".

In the early versions of the F-10 and senior secondary mathematics curricula, uses of technology, where made explicit, were mostly consistent with the *servant* metaphor of Goos et al. (2000), despite the more transformative intentions evident in the initial shaping paper. That is, technology was mostly referred to as an alternative, faster, method for calculating with numbers, manipulating shapes, or drawing graphs. Pedagogical opportunities afforded by the curriculum were restricted to the level of *tasks* in Pierce and Stacey's (2010) taxonomy, in that technology was mentioned in relation to making computation and graphing quicker and more accurate and possibly to link representations. The tasks of the mathematics classroom and the sequencing of topics remained unchanged, despite the potential for technology to support new curriculum goals, new classroom practices, and new ways to treat mathematical topics.

Later Versions of the Curriculum

The General Capabilities section of version 1.2 of the F-10 curriculum maintains the emphasis on digital technologies, stating that "Students develop ICT competencies as they learn to use ICT effectively and appropriately when investigating, creating and communicating ideas and information at school at home, at work and in their communities" (ACARA, 2011, p. 9). This version of the curriculum contains 277 content descriptions, 50 of which (18.1%) refer to technology – a slightly higher proportion than in the consultation version. Nearly half of these descriptions (23 out of 50) refer to performing calculations, creating graphs, or manipulating shapes in the following terms:

> Solve a range of problems involving rates and ratios, *with and without digital technologies* (Year 8 Number and Algebra strand, emphasis added).
>
> Construct displays, including column graphs, dot plots and tables, appropriate for data type, *with and without the use of digital technologies* (Year 5 Statistics and Probability strand, emphasis added).
>
> Investigate the effect of one-step slides and flips *with and without digital technologies* (Year 2 Measurement and Geometry strand, emphasis added).

Saying "with and without digital technologies" instead of "including use of ICT", as in the consultation version, does little to elevate technology above its assumed role as an efficiency tool, equivalent in status and purpose to pencil and paper methods. There is now explicit reference to graphing and geometry software in the Number and Algebra and the Measurement and Geometry strands, but, remarkably, no mention of spreadsheets in the Statistics and Probability content descriptions.

The actual treatment of digital technologies in this version of the curriculum was investigated by electronically searching the document for the terms "technology", "technologies", "calculator", "computer", and "software". Content descriptions containing any of these terms were recorded in tables organised by year level (F-10A) and content sub-strand, for each of the three strands of *Number and Algebra*, *Measurement and Geometry*, and *Statistics and Probability*. The summary below draws out the main findings of this analysis in terms of the roles ascribed to digital technologies in each content strand and for the full range of year levels.

Number and Algebra. There is some reference to technologies in every year level from Years 3 to 10A, mostly in the sub-strands of number and place value, fractions and decimals, and real numbers Technology is mentioned in 21.2% of content descriptions, compared with 19.5% in the consultation version. This is usually in the context of using a range of calculation strategies (mental, written, using appropriate digital technologies), or of performing calculations with and without digital technologies. There is no mention of technology use in the patterns and algebra sub-strand; however, digital technologies and graphing software are highlighted in years 8 to 10A of the sub-strand on linear and non-linear relationships. Examples of their role here include plotting graphs, using graphical techniques to solve linear equations, finding distances, mid-points, and gradients, and exploring connections between algebraic and graphical representations.

Measurement and Geometry. Technology is mentioned in 19.5% of content descriptions, compared with 5.9% in the consultation version. Statements about digital technologies appear in content descriptions for all sub-strands – using units of measurement; shape; geometric reasoning; location and transformation; and Pythagoras and trigonometry – but not for all year levels. Surprisingly, technology is not incorporated into any content statements for Years 8, 9, and 10, despite there being plenty of scope for their use. For example, dynamic geometry software can be used to explore conditions for congruence of plane shapes (Year 8), to use the enlargement transformation to explain similarity (Year 9), or to investigate angle and chord properties of circles (Year 10A). Despite this gap, there is some support for using technology to draw and manipulate shapes, investigate transformations and symmetry, and assist in the development of geometric reasoning.

Statistics and Probability. In the chance sub-strand there is only one reference to digital technologies, in Year 6, in relation to conducting chance experiments. In Years 3, 4, and 5, the data representation and interpretation sub-strand refers to constructing data displays with and without the use of digital technologies. There is no explicit mention of technology in the secondary Years 7, 8, 9, or 10, despite the inclusion, for example, of content statements about comparing data displays, interpreting data displays and the relationship between the median and the mean (Year 7), investigating the effect of outliers on the median and mean (Year 8), and constructing and interpreting boxplots and scatter plots (year 10). Technology is mentioned in only 8.6% of content descriptions, compared with 20.0% in the consultation version. While there is scope for using the internet to address content statements that refer to evaluating statistical reports in the media or collecting data from secondary sources, it is surprising that spreadsheets have not been mentioned as an obvious tool for managing and analysing large data sets.

Technology and Mathematical Practices in the F-10 Australian Curriculum

In terms of the framework proposed by Goos et al. (2000) for describing the role of technology in mathematics teaching and learning, version 1.2 of the F-10 curriculum shows scant evidence of imagining a *partner* role for technology in enabling new approaches to existing tasks for developing students' understanding. The only suggestions for using technology in this way are found in the very few content statements that refer to "investigating", such as:

> *Investigate*, with and without digital technologies, angles on a straight line, angles at a point and vertically opposite angles (Year 6, Measurement and Geometry strand, emphasis added).
>
> *Investigate* combinations of translations, reflections and rotations, with and without the use of digital technologies" (Year 6, Measurement and Geometry strand, emphasis added).
>
> *Investigate* and calculate 'best buys', with and without digital technologies (Year 7, Number and Algebra strand, emphasis added).

By far the majority of statements referring to technology limit its use to that of a *servant* that speeds up, but does not really change, the tasks of the mathematics classroom. Perhaps the real problem is in effectively integrating the proficiency

strands with content descriptions so as to demonstrate how technology can help students develop not only fluency (as implied by the *servant* metaphor), but also understanding, problem solving, and reasoning capacities.

The pedagogical opportunities afforded by the curriculum are still restricted to the level of *tasks* in Pierce and Stacey's (2010) taxonomy, in that teachers are encouraged to use technology to improve speed and accuracy, link mathematical representations, or work with real data. To be fair, it is unrealistic to expect a curriculum document to transform *classroom interactions* (the second level of Pierce and Stacey's framework), since this remains in the realm of pedagogy. Nevertheless, a truly future-oriented mathematics curriculum might make a more serious attempt at transforming the *subject* itself, by (1) supporting curriculum goals that increase emphasis on concepts, applications, and mathematical thinking, or (2) changing the way that mathematical topics are approached and sequenced.

Conclusion

Although the technology messages contained in the *Australian Curriculum: Mathematics* do not do justice to what research tells us about effective teaching and learning of mathematics, it is almost inevitable that there are gaps between an intended curriculum and the curriculum enacted by teachers and students in the classroom. The published curriculum offers many opportunities for creative and effective use of existing digital technologies to teach the required content, whether or not technology is explicitly mentioned in the content statements. Teachers using a future-oriented curriculum should also be able to take advantage of emerging technologies associated with the Web 2.0 paradigm to create dynamic and interactive learning experiences. Within this paradigm, technologies such as wikis afford collaborative knowledge building so that students come to develop genuine expertise rather than relying on the teacher as the only authority. Similarly, portals like YouTube have transformed the Web into a performance medium where students can create and share new mathematical experiences. Immersive, media-rich technology environments therefore have the potential to further transform the mathematical practices of the classroom in ways that are still difficult to imagine. Many teachers are already using technology to enhance students' understanding and enjoyment of mathematics. In their hands lies the task of enacting a mathematics curriculum for the 21st century that will prepare students for intelligent, adaptive, creative, and critical citizenship in a technology-rich world.

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Chapter 8
The Early Years Learning Framework for Australia and the Australian Curriculum: Mathematics – Linking Educators' Practice through Pedagogical Inquiry Questions

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This chapter introduces the *Early Years Learning Framework for Australia* (*EYLF*) and summarises what it has to say about mathematics learning and continuity of learning between preschool and primary school. As well, it discusses the *Australian Curriculum: Mathematics* and its perspective on such continuity of learning. Using a *Numeracy Matrix* developed by the authors with early childhood educators in South Australia, the chapter explores pedagogical links between the *EYLF* and the *Australian Curriculum: Mathematics*. Examples of children's mathematical thinking at both preschool and primary school and of teachers' documentation of this thinking will be used to explore issues of continuity of mathematics learning.

Introduction

The internationally accepted definition of 'early childhood' covers the period from birth to eight years (Organisation for Economic Co-operation and Development (OECD), 2001). This is a time of rapid change, as children grow, develop and learn a great deal about themselves, other people and the world in which they live. Much that young children learn involves mathematics.

Curriculum Frameworks for Mathematics over the Early Childhood Period

For the first time ever in Australia, there are **national** curriculum frameworks that cover the complete span of the early childhood years. For children in prior-to-school settings (mostly aged 0-5 years), there is *Belonging, Being & Becoming: The Early Years Learning Framework for Australia* (EYLF) (Department of Education, Employment and Workplace Relations (DEEWR), 2009), while for children in the early years of primary school (mostly aged 5-8 years), there is the *Australian Curriculum: Mathematics* (AC:M) (Australian Curriculum, Assessment and Reporting Authority

In B. Atweh, M. Goos, R. Jorgensen & D. Siemon, (Eds.). (2012). Engaging the Australian National Curriculum: Mathematics – Perspectives from the Field. Online Publication: Mathematics Education Research Group of Australasia pp. 155-174.

(ACARA), 2011b). Both documents emphasise the importance of children's learning and note some specific outcomes for mathematics learning in the early childhood years. However, each document reflects a different focus on that learning. The EYLF reflects a holistic approach to learning and development, embedded within play-based environments and includes broad learning outcomes. The AC:M is focused on content and proficiency strands, with content descriptions and elaborations.

EYLF outcomes and key components that are pertinent to mathematics learning are a little difficult to find, given the more holistic approach taken and the avoidance of specific learning areas in the document overall. Of most relevance are:

- Outcome 4: *Children are confident and involved learners*, particularly with the two key components:
  - o Children develop dispositions for learning such as curiosity, cooperation, confidence, creativity, commitment, enthusiasm, persistence, imagination and reflexivity; and
  - o Children develop range of skills and processes such as problem solving, inquiry, experimentation, hypothesising, researching and investigating

  and
- Outcome 5: *Children are effective communicators*, particularly the key component:
  - o Children begin to understand how symbols and pattern systems work.

It is also noted in the *Educators' Guide to the Early Years Learning Framework for Australia* (DEEWR, 2010, p. 44) that "Literacy and numeracy capabilities are important aspects of the *Children are effective communicators* Learning Outcome and are vital for successful lifelong learning".

Such outcomes and key components are in distinct contrast to the specific content descriptions in the Foundation Year of the AC:M:

Number and Algebra

- Establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point
- Connect number names, numerals and quantities, including zero, initially up to 10, and then beyond
- Subitise small collections of objects
- Compare, order and make correspondences between collections, initially to 20, and explain reasoning
- Represent practical situations to model addition and sharing
- Sort and classify familiar objects and explain the basis for these classifications. Copy, continue and create patterns with objects and drawings

Measurement and Geometry

- Use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language
- Compare and order the duration of events using the everyday language of time
- Connect days of the week to familiar events and actions
- Sort, describe and name familiar two-dimensional shapes and three-dimensional objects in the environment
- Describe position and movement

Statistics and Probability

- Answer yes/no questions to collect information (derived from ACARA, 2011a).

The four Proficiency Strands: *Understanding, Fluency, Problem Solving* and *Reasoning* are given the following specific foci in the Foundation Year:

- *Understanding* includes connecting names, numerals and quantities ;
- *Fluency* includes counting numbers in sequences readily, continuing patterns, and comparing the lengths of objects directly ;
- *Problem Solving* includes using materials to model authentic problems, sorting objects, using familiar counting sequences to solve unfamiliar problems, and discussing the reasonableness of the answer ;
- *Reasoning* includes explaining comparisons of quantities, creating patterns, and explaining processes for indirect comparison of length (derived ACARA, 2011a).

Young children will move from early childhood programs in prior-to-school settings that are accountable to the philosophies, pedagogies and outcomes of the EYLF to early childhood programs in schools that are accountable to very different philosophies, content and proficiencies in the AC:M. The challenge for all involved is to make this transition into school as successful as possible. One requirement for such success is that the educators involved are able to communicate about the children's prior-to-school mathematics learning and comprehend the significance of this for further learning. This chapter outlines one way in which early childhood educators can link the different pedagogical philosophies and language contained in the EYLF and the AC:M to do this. These links are developed through collaborative interaction based on recognition of, and respect for, the educational value of both prior-to-school and school learning environments.

The Land Between – Transition to School

The EYLF and the AC:M serve different purposes and reflect the different nature of the educational settings for which they are designed. The EYLF makes much of the

underlying principles of secure, respectful and reciprocal relationships in young children's learning and development and the importance of recognising that this learning can be enhanced through partnerships involving families and early childhood educators. The need for high expectations of all in early childhood education, within a context of respect for diversity, imbues the EYLF with an overall social justice approach where all involved are striving for the very best that can be offered for all children. As well, there is an emphasis on early childhood educators as reflective practitioners and ongoing learners.

> A lively culture of professional inquiry is established when early childhood educators and those with whom they work are all involved in an ongoing cycle of review through which current practices are examined, outcomes reviewed and new ideas generated. (DEEWR, 2009, p. 13)

The EYLF mentions the importance of continuity of learning across the transition from prior-to-school years to school years and recognises that different characteristics are likely in schools and prior-to-school settings.

> Transitions, including from home to early childhood settings, between settings, and from early childhood settings to school, offer opportunities and challenges. Different places and spaces have their own purposes, expectations and ways of doing things. Building on children's prior and current experiences helps them to feel secure, confident and connected to familiar people, places, events and understandings. Children, families and early childhood educators all contribute to successful transitions between settings. (DEEWR, 2009, p. 16)

On the other hand, the AC:M does not recognise the existence of the EYLF or, even that children might have learned some mathematics before they come to school. On the contrary, the statement "The early years (5-8 years of age) lay the foundation for learning mathematics" (ACARA, 2011a, p. 6) suggests that a child's mathematical life begins at school entry and that what has occurred to the child beforehand is largely irrelevant. Such a statement would seem to ignore the array of research findings concerning young children's mathematical knowledge well before they start school (Aubrey, 2004; Clarke, Clarke, & Cheeseman, 2006; Ginsburg, 2006; Perry & Dockett, 2005, 2008) and the importance that has been established concerning children's transition to school (Dockett & Perry, 2007; Pianta & Cox, 1999).

In a recent publication, transition to school is defined as

> a dynamic process of continuity and change as children move into the first year of school. The process of transition occurs over time, beginning well before children start school and extending to the point where children and families feel a sense of belonging at school and when educators recognise this sense of belonging. (Educational Transitions and Change (ETC) Research Group, 2011, p. 1)

As they make the transition to school, children establish identities of themselves as learners within the context of school (Dockett & Perry, 2007). A positive start to school is a key factor in promoting children's positive adjustment to, and continued engagement with, school. There is general recognition that transition to school is essentially a process of building relationships and connections, rather than a matter of assessing children's skills or knowledge (Dockett & Perry, 2009). This educational view is sometimes in contrast to the political view, which tends to focus on children becoming 'ready' for school – usually with a focus on literacy and numeracy. However, one reading of the AC:M could be an assumption that what has happened before school is irrelevant anyway. This is contrary to the breadth of research that notes children's competence from birth (Clark & Moss, 2001; Lansdown, 2005).

Many early years educators suggest that what should be sought as children move from prior-to-school settings is continuity of content and pedagogy. Taken together, however, the EYLF and AC:M do not seem to facilitate this and do not really consider what such continuity might be. Continuity of children's learning may well be the ideal. However, this will need to be built by educators as they consider broader issues such as pedagogical continuity, continuity of expectations and experiences, as well as continuity in relationships and support across the prior-to-school and school sectors. There does seem to be agreement that promoting continuity does not mean that contexts should become the same – that is, that prior-to-school and the early years of school become the same. Indeed, there is strong evidence that young children want school and prior-to-school to be quite different; they do not want more of the same as the start school (Dockett & Perry, 2007).

One of the challenges for educators will be to build appropriate continuity. This will involve a great deal of collaboration, cooperation, understanding and commitment from all early childhood educators. Without such efforts, there is a risk of a 'push-down' curriculum in the prior-to-school years and curriculum in the early school years that is disconnected from previous learning. The EYLF and the AC:M, of themselves, do not help facilitate such continuity. The remainder of this chapter reports on one way in which early childhood educators from both prior-to-school and school settings can build such continuity through their own reflective pedagogical practice.

Early Childhood and Powerful Mathematical Ideas

On the surface, prior-to-school settings and first-year-of-school classrooms look quite different. The pre-eminent pedagogical approach in prior-to-school settings is still play, even though there is greater acceptance of other approaches such as sustained shared thinking and, even, some exposition, following the work of the *Effective Provision of Pre-school Education* project (Sylva, Melhuish, Sammons, Siraj-

Blatchford, & Taggart, 2004) which is reflected in the EYLF. While these approaches are also present in first-year-of-school classrooms, there is a much stronger emphasis on more 'formal' pedagogies and situations where assessment *of* learning subsumes assessment *for* learning (Goldstein, 2007).

One of the ways in which some continuity can be discerned in mathematics across prior-to-school and first-year-of-school settings is to consider the powerful mathematical ideas that are experienced by children in these settings. There have been many listings of the key sets of powerful mathematical ideas (Clements, Sarama, & DiBiase, 2004; National Council of Teachers of Mathematics (NCTM), 2000, Perry & Dockett, 2008). By using a set of powerful mathematical ideas, the authors have been able to build an approach which shows promise for assisting educators to bring together the EYLF and the AC:M. The development of this approach is outlined below.

Brief History

The Southern Numeracy Initiative (SNI) was established in South Australia in 2004 with the following aims:

- to develop and implement successful teaching and learning practices to improve numeracy; and
- to challenge teachers to explore their beliefs and understandings about how children develop their understanding of mathematics, and how this can be supported through the teaching program.

Within the broader SNI project, preschool educators developed an important aid to their reflective practice in mathematics: the *Numeracy Matrix*. This was a large table (56 cells) in which seven powerful mathematical ideas derived from a variety of sources lie on one axis (Department of Education, Training and Employment (DETE), 2001, NCTM, 2000, Perry & Dockett, 2002):

- mathematisation;
- connections;
- argumentation;
- number sense and mental computation;
- algebraic reasoning;
- spatial and geometric thinking; and
- data and probability sense;

Eight developmental learning outcomes (DLOs) used in the South Australian preschool curriculum lie on the other axis:

- Children develop trust and confidence;
- Children develop a positive sense of self and a confident and personal group identity;

- Children develop a sense of being connected with others and their worlds;
- Children are intellectually inquisitive;
- Children develop a range of thinking skills;
- Children are effective communicators;
- Children develop a sense of physical wellbeing; and
- Children develop a range of physical competencies (DETE, 2001)

These last outcomes are brought together through 'pedagogical inquiry questions'—questions asked of early childhood educators about the practices they use to assist their children attain both the powerful mathematical ideas and the DLOs. The pedagogical inquiry questions are inputs to the teaching/learning endeavour designed to assist the educators to plan and assess children's learning in both dimensions. This use of pedagogical inquiry questions, rather than student outcome statements, arises from the belief that the key determinants of children's successful outcomes are the pedagogical relationships and practices of educators with the children (Laevers & Heylen, 2004). Table 1 provides an example of one cell in the original numeracy matrix.

Table 1. Numeracy matrix cell

	DLO: Children are intellectually inquisitive
Powerful mathematical idea: *Argumentation*	What opportunities do we give children to put forward a mathematical argument and to justify it? How do we assist children to gain confidence in their ability to explore, hypothesise and make appropriate choices in their mathematics?

In this cell are two pedagogical inquiry questions designed to challenge early childhood educators to reflect on what they are doing to help children develop both the powerful mathematical idea and the developmental learning outcome. Further details about the initial iterations of the *Numeracy Matrix* can be found in Perry, Dockett, and Harley (2007b) and Perry, Dockett, Harley, and Hentschke (2006).

The *Numeracy Matrix* has been used in various forms to facilitate a number of early childhood mathematics initiatives in South Australia since its initial development through the SNI (Perry, 2011; Perry, Dockett, & Harley, 2007a). For example, the pedagogical inquiry questions in the matrix have been linked to narrative assessment of mathematics learning through 'learning stories' (Carr, 2001; Perry et al., 2007a), with the result that children's mathematical knowledge is celebrated and teachers can link their assessment and planning for individual children in meaningful ways. The effectiveness of these links has been identified by educators:

> The numeracy matrix allowed me to identify and extend mathematical learnings occurring in children's everyday experiences.
>
> Learning stories are a really powerful way of collecting, sharing, presenting and reflecting upon children's mathematical learning. I've enjoyed the opportunity to experiment and develop different ways of creating learning stories. (Perry et al., 2007b, p. 130)

Using the *Numeracy* Matrix to Link the EYLF and the AC:M

The possibility that a variation on the *Numeracy Matrix* might be used to help link the EYLF and the AC:M was first addressed in 2010 (Perry, 2011). It was noted that there were strong similarities between the DLOs and the EYLF outcomes (Table 2). As well, the powerful mathematical ideas used to develop the various versions of the *Numeracy Matrix* were recognised in the EYLF:

> Spatial sense, structure and pattern, number, measurement, data, argumentation, connections and exploring the world mathematically are the powerful mathematical ideas children need to become numerate. (DEEWR, 2009, p. 38)

Analysis of the descriptors of the powerful mathematical ideas used earlier (Perry & Dockett, 2008) and the three content strands and four proficiencies of the AC:M provide the mapping shown in Table 3.

Table 2. Relationship between EYLF outcomes and SACSA Developmental Learning Outcomes

EYLF Outcome	Developmental Learning Outcomes
Children have a strong sense of identity	Children develop trust and confidence Children develop a positive sense of self and a confident and personal group identity
Children are connected with and contribute to their world	Children develop a sense of being connected with others and their worlds
Children have a strong sense of wellbeing	Children develop a sense of physical well being Children develop a range of physical competencies
Children are confident and involved learners	Children are intellectually inquisitive Children develop a range of thinking skills
Children are effective communicators	Children are effective communicators

Table 3. Relationship between original powerful mathematical ideas and AC:M content strands and proficiencies

AC:M content strands and proficiencies	Powerful mathematical ideas
Number and Algebra	Number sense and mental computation Algebraic reasoning
Measurement and Geometry	Number sense and mental computation Spatial and geometric thinking
Statistics and Probability	Data and probability sense
Understanding	Connections Mathematisation
Fluency	Connections Mathematisation
Problem Solving	Mathematisation Argumentation
Reasoning	Argumentation

Adaptation of the *Numeracy Matrix* to reflect the EYLF outcomes and the AC:M strands and proficiencies has generated a mechanism for linking the two national curriculum documents (Table 4[1]). The use of pedagogical inquiry questions within the revised *Numeracy Matrix* means that early childhood educators can be asking themselves the same questions about their own pedagogical practice but answering them through links to whichever of the two curriculum documents pertains to their setting. If early childhood educators reflect on their mathematical pedagogies using the inquiry questions, they can use these, in conjunction with learning stories or other documentation and planning tools, to provide some continuity between mathematics learning in prior-to-school and school settings. However, this process achieves a lot more than this. It also provides early childhood educators with a language through which they can communicate with their colleagues across the pedagogical divide between prior-to-school and school services: a divide which is not helped by the substantial differences in the EYLF and the AC:M.

[1] *The authors are grateful to the expertise of Di Hogg who was a major contributor to the development of this version of the Numeracy Matrix.*

Table 4. Numeracy Matrix linking EYLF and the AC:M

EYLF Outcomes/ AC:M Strands and Proficiencies	**Number and Algebra**	**Measure-ment and Geometry**	**Statistics and Probability**	**Under-standing**	**Fluency**	**Problem Solving**	**Reasoning**
OUTCOME 1 **Children have a strong sense of identity**	What opportunities do we provide for children to seek new challenges and persist in their problem solving? In what ways do we assist children to represent varied physical activities and games through patterns and symbols? How do we encourage children to explore different perspectives in mathematical problem solving?	How do we encourage children to work collaborat-ively with peers during measure-ment activities? In what ways are children able to demon-strate flexibility and make choices when playing with collections of everyday shapes and objects?	How do we encourage children to develop a notion of fairness in their lives?	How do we encourage children to play and interact purposefully with the mathematics they experience in their lives? What do we do to assist children link real world represent-ations of mathematics with their own mathematical words and symbols?	How do we encourage children to use different communi-cation strategies to organise and clarify their math-ematical thinking? What opportune-ities do children have to show their patterning abilities?	How do we encourage children to use the process of play, reflection and investiga-tion to solve math-ematical problems? How do we assist children to gain confidence in their ability to explore, hypoth-esise and make appropriate choices in their math-ematics?	How do we encourage children to demonstrate flexibility and to manage different mathematical ideas as they are presented to them by peers?

EYLF Outcomes/ AC:M Strands and Proficiencies	**Number and Algebra**	**Measure-ment and Geometry**	**Statistics and Probability**	**Under-standing**	**Fluency**	**Problem Solving**	**Reasoning**
OUTCOME 2 **Children are connected with and contribute to their world**	In what ways do we establish an environment that promotes children's exploration? How do we encourage children to explore patterns? In what ways do we provide opportunities for children to reflect upon their mathematical pattern making? What opportunities do we provide for children to feel confident in their under-standing of symbols and patterns in their environment?	What opportunities do we provide for children to see the purpose of measurement in their world? How do we provide opportunities for children to apply and challenge using a variety of measurement experiences? In what ways do we encourage children to explore relationships among collections of shapes? What opportunities do we provide for children to develop awareness of similarities and differences among shapes and objects?	How do we assist children to gather information, ask questions, seek clarification and consider possibilities about their own lives?	What opportunities do we provide for children to reflect upon and respect diversity and connections between people's mathematical knowledge and strategies? How do we encourage children to see the mathematics in their worlds and to use that to develop new mathematical ideas?	What opportunities do we provide for each child to demonstrate enthusiasm for new mathematical tasks? What opportunities do we provide for children to connect different mathematical ideas they learn?	What opportunities do we provide for children to explore different perspectives as they attempt to solve mathematical problems? How do we encourage children to help develop and maintain agreed values and socio-mathematical norms of behaviour in their groups?	How do we encourage children to integrate their mathematical thinking with their commun-ication skills so that they can justify their opinions? What opportunities do we provide for children compare and contrast their ideas with those of other children?

EYLF Outcomes/ AC:M Strands and Proficiencies	**Number and Algebra**	**Measurement and Geometry**	**Statistics and Probability**	**Under-standing**	**Fluency**	**Problem Solving**	**Reasoning**
OUTCOME 3 **Children have a strong sense of wellbeing**	What opportunities and support do we give children to take risks when developing under-standings about number? How do we encourage children to generate a range of ideas and to use the processes of play, reflection and investigation to find answers to problems?	How do we give children opportunities to expand their measurement language? How do we use play to develop opportunities for children to explore measurement? What opportunities do we provide for children to make discoveries that are new to them about shape and space? How do we encourage children to move confidently in space and perform different movement patterns with growing spatial awareness?	How do we encourage children to make choices in their lives? What opportunities do we provide for children to predict and manage change in their daily routines?	How do we encourage children to take risks as they seek to find the mathematics in everyday life? What opportunities do we provide for children to use mathematics to help predict and manage change in their daily lives?	What do we do to assist children experience success in their mathematics learning? What opportunities doe we provide for children to investigate the mathematical aspects of their cultures?	How do we encourage children to interact with others to explore ideas, negotiate possible solutions and share their mathematical learning? What skills do we develop to assist children undertake mathematical problem solving in pairs or in larger groups?	What opportunities do we provide for children to develop confidence in expressing their mathematical ideas? How do we provide the best possible environment in which children can create and synthesise using mathematics?

EYLF Outcomes/ AC:M Strands and Proficiencies	Number and Algebra	Measure-ment and Geometry	Statistics and Probability	Under-standing	Fluency	Problem Solving	Reasoning
OUTCOME 4 **Children are confident and involved learners**	What opportunities do we provide for each child to accept new challenges, make new discoveries and celebrate effort and achievement? What do we do to encourage children to use symbols and different representations of their mathematics?	How do we ensure that children have confidence to access and use resources for measurement? How do I support children to choose what to measure, how to measure and how to represent their measurement? What opportunities do we give children to explore their local environment and record what they see using visual means?	How do we support children to explore groups to which they belong, based on particular attributes? What opportunities do we provide for children to explore the ideas and concepts of data representation?	How do we encourage children to actively explore mathematical problems and investigate relevant problems through mathematics? What opportunities do we provide for children to connect their mathematical ideas?	How do we encourage children to use mathematics to be a critical consumer of everyday products? How do we encourage children to move confidently in space and perform different movement patterns with growing spatial awareness?	How do we encourage children to use technology to help them solve mathematical problems? What do we do to assist children develop persistence in their mathematical problem solving?	How do we encourage children to question why their and other people's mathematical ideas work? How do we encourage children to seek more than one answer to a mathematical problem?

EYLF Outcomes/ AC:M Strands and Proficiencies	Number and Algebra	Measure-ment and Geometry	Statistics and Probability	Under-standing	Fluency	Problem Solving	Reasoning
OUTCOME 5 **Children are effective communicators**	What opportunities do we give children to explore, hypothesise, take risks and engage in symbolic and dramatic play with confidence? How do we encourage children to represent number in a variety of ways? How do we encourage children to talk about and represent their findings? How do we encourage children to demonstrate an under-standing that symbols are a powerful means of communica-tion?	How do we assist children to use pattern making and pattern continuation for problem solving and investigation? How do we encourage children to participate in group discussions and brainstorms around the properties of shapes? How do we encourage children to use different communication strategies to describe shapes and their properties?	How do we encourage children to collect, analyse and represent data about their physical activity? How do we encourage children to begin to recognise, discuss and challenge unfair attitudes and actions?	How do we encourage children to gather information and ask questions that might be answered by this information? What opportunities do we give children to describe their mathematical thinking?	How do we encourage children to contribute to collaborative group work in mathematics through taking on a variety of roles?	How do we encourage and support children to talk about and represent their efforts to solve mathematical problems? What opportunities do we give children to document their solutions to mathematical problems?	How do we encourage children to participate in group discussion and justification about the solution of mathematical problems?

As with all previous iterations of the *Numeracy Matrix*, this one should be seen as a work in progress. The pedagogical inquiry questions are meant to be relevant to the particular contexts in which early childhood educators find themselves. While those in Table 4 have been developed by educators, they should not be seen as fixed. Educators are encouraged to develop pedagogical inquiry questions that reflect their own contexts and the children within these.

Early childhood educators in schools and prior-to-school settings can reflect on their practices through the pedagogical inquiry questions which can be attached to either the EYLF or the AC:M, depending on whether the educators focus on the first column of the matrix or the first row. Using the matrix, a prior-to-school educator could communicate with a school educator during a child's transition to school to highlight what the child did in response to, for example, *encouragement to use symbols and different representations of their mathematics* (EYLF Outcome 4 and AC:M Number and Algebra). Using this pedagogical inquiry question as a starting point the two educators could discuss the child's response either from the perspective of the child being a confident or involved learner or from the perspective of the child learning particular number and algebra ideas. If the previous versions of the *Numeracy Matrix* can be taken as a guide (Perry et al., 2007a; Perry, 2011), the most likely is that educators will talk about both the outcome and the strand, setting up a situation which recognizes children's prior, current and future learning.

Using the *Numeracy Matrix*

In this section of the chapter, two learning stories are presented to illustrate how sensitive and reflective educators can use the pedagogical inquiry questions integral to the *Numeracy Matrix* to assist them in assessing and planning children's mathematical activities. Learning stories are qualitative snapshots, recorded as structured written narratives, often with accompanying photographs that document and communicate the context and complexity of children's learning (Carr, 2001). They include relationships, dispositions and an interpretation by someone who knows the child well. They are "structured observations in everyday or 'authentic' settings, designed to provide a cumulative series of snapshots" (Carr & Claxton, 2002, p. 22).

The first learning story presented here has been written by a preschool educator working in a rural area of South Australia. The second has been written by a teaching assistant in a remote school in South Australia. The authors of this chapter are very grateful to these educators for allowing the learning stories to be reproduced here. Confidentiality requirements mean that neither the educators nor the children involved can be named (pseudonyms are used for the children).

Samson[2] knows a BIG number

Samson was sitting playing at the play dough table and I went to have a chat with him about making a number game. Samson said to me, 'I know a really big number – a million.' I asked Samson if he knew how to write one million in numerals and, as he didn't, I showed him. After briefly looking at the numbers I had written down (1,000,000) Samson said, 'Now I get it, a million is six zeros. A thousand is a one and three zeros. A hundred, one and two zeros. If you took three zeros away (from a million) it would be a thousand.' I asked Samson what the number would be if I replaced the one with a six and he told me it would be six million. Samson then said that he knew an even bigger number, a fillion! I said that there was not a number called a fillion but there was a billion (with nine zeros) and a trillion (with twelve zeros). He was very impressed by the number of zeros in these numbers.

Follow up

Following our conversation Samson decided to paint a picture. He painted numbers from zero to fourteen on his paper. I asked Samson why he had stopped at number fourteen and he said that fourteen was his favourite number, he just likes the four.

Figure 1. Samson knows a BIG number

A preschool educator might link this learning story particularly to the EYLF Outcome 4: *Children are confident and involved learners*, although it would be possible to link it with other EYLF outcomes. When speaking with a first-year–of-school educator about Samson, the preschool educator might concentrate on the confidence Samson has shown. The school teacher will be interested in this confidence as well but will also focus on the number knowledge being displayed. Hence, this learning story could bring the two educators together through the *Numeracy Matrix* cell shown in Table 5.

Table 5.Numeracy matrix cell

	Number and Algebra
OUTCOME 4 **Children are confident and involved learners**	What opportunities do we provide for each child to accept new challenges, make new discoveries and celebrate effort and achievement? What do we do to encourage children to use symbols and different representations of their mathematics?

Both of these pedagogical inquiry questions have relevance to Samson's learning story. Opportunities have been provided for the child to seek the pattern involved in writing the standard symbolic representations of big numbers and to celebrate

[2] *Samson is 4 years old*

the effort and achievement resulting from this opportunity. The activity created a challenge for the child and Samson was encouraged to accept the challenge of even bigger numbers. In fact, he set himself this challenge, confident in his previous experience with big numbers. The educator has assessed Samson's understanding of the patterns that he is developing and shown that his pattering seems to be robust.

While Samson and the educator are working simultaneously on **Number and Algebra** and **are confident and involved learners**, the emphasis will be on the EYLF outcome for the preschool educator. A first-year-of-school educator about to receive Samson into her classroom might be more interested in how this knowledge of numbers, number symbols and patterns could be extended even further. Nonetheless, using the pedagogical inquiry questions in this cell of the *Numeracy Matrix*, the two educators can discuss what opportunities the child already has had and what opportunities he might be given when he gets to primary school, thus facilitating some continuity of pedagogy which might otherwise have been missing. This will mean that not only will the school educator know what the child knows about big numbers but will also have some ideas about how the child knows these things and how he likes to learn about them. As well, the learning story tells both educators that while Samson seems to have substantial knowledge about big numbers and the ways that they are recorded, he also likes to revert to his safe and familiar spaces, represented by his 'favourite' number.

The long and short of it

Harry[3], today you played Red Rover on the tennis courts. Everyone lined up and the caller called over people wearing shorts. You looked down at your legs and seemed unsure as to whether you were wearing shorts or long pants. When you bent forward, your shorts got longer; it was a bit of a puzzle.

You turned to the person next to you and compared what they were wearing with what you were wearing.

You looked back and forward from their legs to yours. Your friend said, "go on Harry, you've got shorts on."

You decided that you were indeed wearing shorts and took off across the court.

The next time that the caller called for people wearing shorts, you had no hesitation. You took off across the court and ran so fast that nobody could catch you.

What was the learning?

Harry today you were presented with a puzzle, were you wearing long shorts or short longs, and why did they get longer every time you leant forward?

I saw you look for a solution to the puzzle by comparing what you had on with what your friend had on. Your friend offered you an answer, and you showed respect for their opinion by accepting it as the answer. The next time the caller wanted people wearing shorts to cross the court; you remembered what you had learnt earlier and took off without hesitation.

What's next?

We can support your problem solving skills by offering you a variety of experiences that challenge your thinking such as computer games, matching puzzles and math and science experiments.

Figure 2. The long and short of it

[3] *Harry is 5 years old.*

This learning story highlights the importance of children's observations and language as they explore measurement challenges they meet in everyday life. The keen observation of the educator almost begs for further discussion with Harry around what happens to his shorts as he leans forward. This discussion will inevitably involve the language of mathematics and will encourage the development of stronger comparison skills. The *Numeracy Matrix* provides other opportunities for educators to analyse the current activity and to plan future tasks. For example, Table 6 offers four pedagogical inquiry questions of relevance to Harry's scenario.

Table 6. Pedagogical inquiry questions from several Numeracy matrix cells

	Measurement and Geometry	Understanding
OUTCOME 2 **Children are connected with and contribute to their world**	What opportunities do we provide for children to see the purpose of measurement in their world?	How do we encourage children to see the mathematics in their worlds and to use that to develop new mathematical ideas?
OUTCOME 3 **Children have a strong sense of wellbeing**	How do we use play to develop opportunities for children to explore measurement?	How do we encourage children to take risks as they seek to find the mathematics in everyday life?

In terms of the reporting of Harry's perplexing situation – at least perceived as perplexing by the educator – he was certainly in a situation where measurement helped him understand his world and where he was able to explore measurement ideas whilst playing. He perceived his shorts from a number of perspectives as he bent his body and he also dealt with his friend's perspective on the dilemma. All of this required him to portray his dilemma in terms of mathematics. In terms of planning for future tasks, the educator has suggested supporting his explorations of measurement skills and development of his understandings through a number of activities in which there are elements of 'controlled' risk, allowing Harry some control without endangering his emerging willingness to engage in risk-taking behaviours in measurement. While prior-to-school and school educators might analyse this learning story from different perspectives, the pedagogical inquiry questions linking the two curriculum documents provide a common language through which pedagogical continuity can be generated.

Conclusion

The benefits for children of a positive start to primary school are well documented (Dockett & Perry, 2007; Dunlop & Fabian, 2007; Peters, 2010; Pianta & Cox, 1999). Part of a successful start to school is recognition that young children often bring a great deal of mathematical knowledge and many dispositions towards mathematics learning with them. Some of this knowledge is drawn from the "canonical curriculum for school mathematics" (Nebres, 1987 cited in Perry & Conroy, 1994, pp. 87) but much of it has been learned from everyday experiences.

For the first time, Australia has national curriculum documents that will be used in all prior-to-school and school settings. The two documents have the potential to impact critically on young children's mathematics education. The transition to school period will be very important in ensuring that the mathematics learning that children have accomplished partly under the auspices of the EYLF is not missed by the AC:M with its more formalised approach to young children's learning. On the other hand, the transition to school will be very important in ensuring that young children are not disadvantaged as a result of their learning inspired by the EYLF when they meet the learning inspired by the AC:M. The chapter has reported on a strategy through which the two national curriculum documents covering early childhood mathematics education in Australia can be used to support recognition of young children's mathematics knowledge and understanding in both contexts. The introduction of the *Numeracy Matrix* in South Australian preschools and schools has enabled educators in both sectors to use common language and constructs in order to communicate about mathematics learning. The *Numeracy Matrix* may provide one way in which the two curriculum documents might be brought together to enable young children to start school successfully, at least in terms of their mathematics education. It also promotes recognition of the important role of educators in prior-to-school and school settings as catalysts for young children's mathematics learning, as it highlights the centrality of their actions and interactions with children, families and each other.

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Chapter 9
The Australian Curriculum: Mathematics as an Opportunity to Support Teachers and Improve Student Learning

Peter Sullivan

The creation of a national, as distinct from state and territory based, mathematics curriculum creates important opportunities for improving learning, but whether those opportunities are taken up will depend on the ways that teachers are supported, including by teacher educators, coaches, school leaders and readers of this monograph. The debates that continue on aspects of content are irrelevant to whether the national curriculum provides a prompt to improvement. In fact, such debates are a by-product of the negotiations that are an obvious artefact of the ceding of responsibilities by local jurisdictions to a national authority. The real opportunities for improving mathematics learning are in the principles that underpin the structure of the curriculum and the use of these principles to inform teacher learning.

Introduction

This Chapter describes the potential of aspects of the Australian Curriculum: Mathematics (AC:M) to prompt reconsideration of current approaches to teaching and learning mathematics. In particular, three principles on which the curriculum is based are that:

- the four proficiencies (understanding, fluency, problem solving and reasoning) provide a clearer framework for mathematical processes than "working mathematically" and are more likely to encourage teachers and others who assess student learning to move beyond a focus on fluency, however, there will need to be support for teachers if they are to incorporate them into the curriculum;
- the curriculum has been written to emphasise teacher decision making, therefore, curriculum support for teachers should be in a form that enhances their decision making, rather than reduces it; and
- the challenge of equity can be addressed by focusing on depth of learning rather than breadth, by specifically supporting the learning of those students who need it and by extending more advanced students within the content for that level rather than isolating such students into different classes.

This chapter elaborates the rationale for each of these three principles, illustrates how these are represented in the curriculum, and proposes particular emphases for

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associated teacher learning. The chapter uses an extract of the content related to the teaching of fractions and an illustrative teaching activity to elaborate the discussions of each of these issues.

The Presentation of the Content Descriptions

The following extract from the AC:M is used to illustrate the way the curriculum is presented and to facilitate later discussion of the above principles. The descriptions presented are those related to the teaching of fractions in the middle years. There are other descriptions related to decimals, percentages and ratios that are obviously closely related although these are not presented here.

Year 5

Compare and order common unit fractions and locate and represent them on a number line

Investigate strategies to solve problems involving addition and subtraction of fractions with the same denominator

Year 6

Compare fractions with related denominators and locate and represent them on a number line

Solve problems involving addition and subtraction of fractions with the same or related denominators

Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies

Year 7

Compare fractions using equivalence

Locate and represent fractions and mixed numerals on a number line

Solve problems involving addition and subtraction of fractions, including those with unrelated denominators

Multiply and divide fractions and decimals using efficient written strategies and digital technologies

Recognise and solve problems involving simple ratios

Some obvious features are that, in comparison to most other Australian jurisdictional curricula, the descriptions are clear, concisely worded, and few in number. The sequential development is obvious as is the scope of what it is expected that students will learn. As is described below, this is a desirable and potentially productive aspect of the curriculum.

A further aspect of the content descriptions is the way that they are presented. Being set in a flexible web format (Australian Curriculum Assessment and Reporting Authority, ACARA, 2011), teachers can have the content descriptions displayed by year level, by substrand across year levels, or even in comparison with other subjects. This means that, however teachers like to use curriculum documents as part of their planning, the web format can support this. There is no need for the development of school based cut-and-paste versions or extracts of the curriculum, and so teachers can focus on the real issues of planning their teaching and assessment. The effective use of the potential of the web based format can be an important element of teacher professional development.

Identifying the Nature of the Mathematics and Numeracy we want Australian Children to Learn

Any discussion of curriculum is contingent on a view of what mathematics the curriculum is seeking to present. The AC:M takes an explicit stance that the mathematics and numeracy that should be experienced by school students is much more than the emphasis on procedures and computational processes that seems to constitute much current teaching of mathematics in Australia (see Hollingsworth, Lokan, & McCrae, 2003; Stacey, 2010). In various places in this chapter a particular task is used to elaborate the arguments presented, and is also used to represent a particular perspective on mathematics. The task is:

> Ano can paint a house in 3 days. Elizabeth can paint a house in 4 days. How long would it take Ano and Elizabeth to paint a house if they worked together?

Depending of the approach used, this task addresses particular aspects of the content descriptions described above. Indeed the task is useful for teaching junior secondary mathematics precisely because there is a range of possible approaches and so not only allows consideration of various approaches but also can be used to illustrate the connections between them. Some of the possible approaches are described in the following. One mathematically correct approach is as follows:

> Ano can paint $\frac{1}{3}$ of a house in one day, and Elizabeth can paint $\frac{1}{4}$ of a house in a day. So in one day they could paint $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$ of the house. So they could paint the whole house in $\frac{12}{7}$ or $1\frac{5}{7}$ days.

This approach addresses some of the content descriptions associated with Year 7 including problem solving, addition of fractions and division of fractions. There would be substantial challenge for the students to explain why the fraction is inverted to convert from time per house to house per day but otherwise the explanation of the process is straightforward. Another approach which produces the precise answer, this time involving the use of ratios is as follows:

> Ano could paint 4 houses in 12 days. Elizabeth could paint 3 houses in 12 days. So together they could paint 7 houses in 12 days. Which means they could take $\frac{12}{7}$ days to paint one house.

This approach requires some insight to develop but is easier to understand and possibly more generalisable to other related situations. It allows the teacher to make the connection between fractions and ratios. Another approach might be:

> Ano can paint half a house in one and one half days, and Elizabeth can paint half a house in 2 days. So the total time needed is more than one and one half days but less than two days. Assuming that once Ano finished her half she helped Elizabeth, it is reasonable to suggest that they will take around $1\frac{3}{4}$ days.

This is not the precise answer but is quite close and represents both clear and appropriate thinking about fractions and their applications and also illustrates the type of approximations that are made in realistic situations. There can even be discussion of how to evaluate the limitations of the approximations. Another approach requiring approximations could be as follows:

> Ano can paint $\frac{1}{3}$ of a house in one day, and Elizabeth can paint $\frac{1}{4}$ of a house in a day. So in one day they could paint $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$ of the house. After two days they could paint $\frac{14}{12}$ of the house which is more than is needed by $\frac{2}{12}$ or $\frac{1}{6}$. So Ano and Elizabeth can paint the house together in $1\frac{5}{6}$ days.

This approach also illustrates some clear thinking about fractions, intelligent reasoning, and produces an approximation of the precise answer. Of course, the inaccuracy arises because it is $\frac{1}{6}$ of a house too much, not $\frac{1}{6}$ of a day but even this allows teachers to explore such misconceptions.

This type of task represents a different sort of mathematics from the common approach in which fractions are taught using rules and procedures and where examples are presented without any context. Of course, it can be argued that this task is unrealistic but the task allows the consideration of a model of fraction manipulation that the students at this level can relate to. There are practical examples that do require this type of thinking but they mainly relate to continuous quantities that increase the complexity of the situation beyond what is appropriate for an introductory exercise. Solutions to the task require some assumptions to be made. For example, most of the above suggestions assume that all houses take the same time to paint. This leads to further opportunities in challenging those assumptions.

Assuming that the teacher draws on the various strategies used by the students, the experience will communicate to students that there are many ways to approach mathematical tasks, that they can choose their own approach (which is motivational – see Middleton, 1995), that some approaches are more efficient than others, that sometimes there is a mathematical and a practical answer, that mathematics and fractions can have practical meanings, and that sometimes it is necessary to consider different aspects of a task simultaneously. There is also potential that such tasks can expose student misconceptions which can then be addressed by the teacher in the context of the task that was posed.

In other words, the task and the associated pedagogies allow students to see that mathematics, and in this case fractions, is more than following rules and procedures but can be about creating connections, developing strategies, effective communication, and so on. While this view is not obvious in the content descriptions presented above, it is part of the opportunity for those supporting teachers to communicate such views. The view is, though, communicated through the proficiencies that underpin the curriculum as is described in the next section.

The Four Proficiencies are Better for Presenting Mathematical Actions than "Working Mathematically"

ACARA (2010) proposed that the *content* be arranged in three strands that can be thought of as nouns, and four *proficiency* strands that can be thought of as verbs. The content strands, *Number and Algebra*; *Measurement and Geometry*; and *Statistics and Probability*, represent a conventional statement of the "nouns" that are the focus of the curriculum. The content descriptions presented above and the painting task are from the Number and Algebra strand.

More interesting, and a break from the common ways of describing the mathematical actions in which we hope that students will engage, are the four proficiency or process strands which were adapted from the recommendations in *Adding it up* (Kilpatrick, Swafford, & Findell, 2001). The first of these is "Understanding" (the Kilpatrick et al. term was *conceptual understanding*) and is described in the AC:M as follows:

> Students build a robust knowledge of adaptable and transferable mathematical concepts, they make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the 'why' and the 'how' of mathematics. Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically and when they interpret mathematical information.

Understanding has long been a goal and teachers are familiar with its importance. Skemp (1976), for example, explained that it is not enough for students

to understand how to perform various mathematical tasks; they must also appreciate why each of the ideas and relationships work the way that they do.

In the painting task, students can build their understanding by making connections between related concepts, by seeing fractions and ratios as interchangeable, and by representing and seeing representations of the same information in different ways. Representing and recording their thinking using language and symbols also connects to understanding. When solving the task, students can use the symbols, words, and relationships associated with the particular concepts, connect these different representations to each other and use them later in building new ideas.

In the content descriptions above, the verbs such compare, order, locate, and represent, are associated with understanding.

The second of the proficiencies is fluency (the Kilpatrick et al. term was *procedural fluency*) includes:

> ... choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, when they recall definitions and regularly used facts, and when they can manipulate expressions and equations to find solutions.

Watson and Sullivan (2008) used term *mathematical fluency* to include skill in carrying out procedures flexibly, accurately, efficiently, and appropriately, and, in addition to these procedures, having factual knowledge and concepts that come to mind readily. While this is the aspect of mathematics learning that is most commonly emphasised in much mathematics teaching and assessment, there is a clear rationale for fostering appropriate levels of fluency for students. This in part relates to cognitive load theory (see Sweller, 1994; Bransford, Brown, & Cocking, 1999). Pegg (2010) explained that initial processing of information happens in working memory, which is of limited capacity and therefore being able to recall relevant information from long term memory assists people's capacity to process and use information. In the case of the painting task, fluency refers to the efficient manipulation of the numbers, such as addition of the fractions or manipulating the ratios (e.g., one house in 3 days, is $\frac{1}{3}$ of a house in one day), choosing efficient strategies, and awareness of ways of dealing with fractions.

Examples of the actions suggested in the above content descriptions that prompt consideration of fluency are locate, represent, multiply and divide using efficient written strategies.

The third of these mathematical actions is problem solving (*strategic competence*) which was described as:

> ...the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify their answers are reasonable.

Problem solving has been a focus of research, curriculum and teaching for some time, and so teachers are familiar with its meaning and resources that can be used to support students learning to solve problems. In the case of problem solving, there are opportunities in the painting task for students to use more or less all of the actions included in this problem solving statement either in formulating their own solution or in listening to the solutions of others.

As an aside, the problem solving proficiency also creates opportunities to connect the mathematics that students are learning to their lives. Even though there are many interesting problems that have the status of puzzles, there are also many opportunities to use mathematics in interesting ways to understand practical situations that are not only those connected to the world of work or future study but also those that create more informed citizens.

The phrase "solve problems" is mentioned four times in just the few content descriptions above indicating that even with the learning of introductory fractions the intent is that students solve problems as a pathway to that learning.

The fourth proficiency, reasoning (*adaptive reasoning*) includes:

> ...analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false and when they compare and contrast related ideas and explain their choices.

As with problem solving, students have the opportunity to experience more or less all of the actions described as reasoning either in their own thinking about the painting task or in listening to the explanations or arguments of other students and the teacher. As Stacey (2010) argued that there are few opportunities for student initiated reasoning in most current Australian texts, and it is suspected that this is the proficiency that is least familiar to Australian teachers.

As it happens, and perhaps unfortunately, reasoning is explicitly prompted only by the term *investigate* in the content descriptions above, although the intent of the overall proficiencies and the achievement standards is that reasoning be emphasised in the teaching of mathematics.

It is suspected that teachers will need support on how student reasoning applies to experiences with the foundational and other ideas of the curriculum. It is possible, for example, for students to explore and formulate hypotheses about partitioning numbers, perimeter and area relationships, effective definitions of shapes, comparing graphical representations, and then to explain their thinking, justify their argument, convince others, listen and interpret the arguments of others, and so on. Fostering reasoning through the mainstream content requires a teaching stance where problems with possibilities of student decision making and choice are posed for the students to explore, the lessons and the classrooms are structured to facilitate communication both while students are working on the task and then during whole class discussion reviewing the task, with an expectation that errors are learning opportunities and diversity of approaches are valued.

At least part of the reason for adopting the proficiencies rather than "working mathematically" as the prompt or context for encouraging teachers to plan their teaching in such a way as to foster these actions by students is that the ways that "working mathematically" was presented to teachers by many jurisdictions created the impression that it was an additional content strand. It was common for teachers to plan their teaching by addressing the topic of Number for a few weeks, then Space for a few weeks, then Working Mathematically at the end of term. The intention in the AC:M is that the proficiencies apply to all aspects of mathematics. The metaphor of verbs acting on nouns describes the explicit intention to ensure that the emphasis is on the full range of mathematical actions and not just fluency. The challenge for teachers is to find ways to incorporate a balance of these different verbs in their teaching.

The integration of the content descriptions and the proficiencies is represented by an icon that shows the three content strands in one direction and the four proficiencies in the other. Interestingly similar icons are presented for each of the four curriculum subjects (English, science and history as well) indicating that the need to integrate content and action dimensions of curriculum applies more broadly than just to mathematics.

The use of four proficiencies also emphasises to teachers that doing mathematics is more than procedural fluency.

The Kilpatrick et al. terms have been slightly simplified for ease of communication, and the proposed words are in common usage among teachers in Australia. Those familiar with the Kilpatrick et al. report will notice that "productive disposition" is not included in this list. One reason is that disposition was taken to refer to pedagogical approaches which were not proposed to be included in the curriculum statements. Another reason is that the proficiencies are intended to inform assessment and particularly the articulation of standards, and it is neither meaningful not appropriate to set standards for disposition. The omission of disposition is not intended to infer that disposition is less important than other proficiencies. Given that the curriculum was intended to define content and student actions, and that teachers would assess students' enaction of that content and the associated actions, it is clear that disposition is quite different. Nevertheless student disposition is a key consideration for mathematics teachers and its meaning and actions to foster productive disposition can be addressed in teacher learning.

It is noted that, while the first two proficiencies may be able to be pursued by teachers using clear explanations and examples drawn from texts, problem solving and reasoning require the selection of appropriately structured tasks that allow students opportunities to make decisions, to choose their own approaches, and to connect ideas together. Indeed part of the focus of teacher learning associated with these proficiencies should be on ways of identifying tasks that can facilitate student engagement with all four of these proficiencies. It is also noted that the proficiencies act together and so should not necessarily be fostered separately. For example, teachers might pose tasks that require students to solve a practical problem. Students can then be invited to explain the reasoning behind their solution. In this way the teacher can facilitate the building of students' understanding.

Teacher Decision Making

> One of the principles argued in ACARA (2010) is that the curriculum should be described clearly and succinctly. Based on the nature of support that they publish, some Australian jurisdictions seem to believe that teachers are best supported by providing minute by minute guidelines on how teachers should teach. In contrast, the AC:M has taken the stance that the curriculum should be described parsimoniously and presented flexibly via a dynamic web based environment and that this will assist teachers by emphasising the need for them to make active decisions.

It goes without saying, regardless of the educational context, teachers are better able to support students when they know what they hope the students will learn. Hattie and Timperley (2007), for example, reviewed a large range of studies on the characteristics of effective classrooms. They found that feedback was one of the main influences on student achievement. The key elements identified were that students should receive information on "where am I going?", "how am I going?", and "where am I going to next?" To advise the students interactively, it is important for teachers to know their goals.

In the context of the task presented above, whether the teacher started their planning from the content descriptions or the task itself, it is essential that the teacher make active decisions on the goals of learning, the actions in which they want students to engage and how they will assess them. In particular the teachers should know:

- the mathematical point of that task
- the pedagogical point of that task
- whether and how they might make these points explicit to students.

Assuming that the teacher reads the content descriptions at some stage in their planning and that the teacher has a resource of problems on which to draw, an early decision is to choose which tasks are most suited to the relevant content. The next decision is how best to introduce the task to students and how to support them in their learning. For example, in the painting task, the teacher might decide to focus on the problem solving and reasoning proficiencies and so might allow students opportunities to explore pathways to solutions for themselves, and will create time in the lesson for students to explain and justify their strategies, to compare their strategies with others, and to generalise the fractional ideas to other situations.

One of the disadvantages of having the content determined by a text is that teachers are less required to think about their own purposes. The same is true for some curriculums in which the teachers are recommended which tasks to teach without having to appreciate the goals, both content and proficiencies, associated with the tasks. One of the critical foci for teacher learning is to enhance their capacity to make their own decisions using the curriculum documents and the other resources to which they have access.

The Challenge of Equity

> A third key element of the AC:M, and which can be a priority in future teacher learning is ways to address the challenge of equity. In various reports on international assessments (e.g., Thomson, De Bortoli, Nicholas, Hillman, & Buckley, 2010) and in other analyses (e.g., Sullivan, 2011) the diversity of achievement of Australian students is noted. In particular, it seems that low SES students, as a group, perform substantially below other students. This is connected to the curriculum in various ways.

ACARA (2010) argued that all students should experience the full range of mathematics in the compulsory years. Mathematics learning creates employment and study opportunities and all students should have access to these opportunities. This is both an equity and a national productivity issue. The curriculum makes the explicit claim that all students should have access to all of the mathematics in the compulsory years.

> A fundamental educational principle is that schooling should create opportunities for every student. There are two aspects to this. One is the need to ensure that options for every student are preserved as long as possible, given the obvious critical importance of mathematics achievement in providing access to further study and employment and in developing numerate citizens. The second aspect is the differential achievement among particular groups of students. (ACARA, 2010)

Many systems and schools are structured to offer some students a restricted subset of the mathematics curriculum. This is counter-productive. Chris Matthews, the first Indigenous Australian with a doctorate in applied mathematics, in his presentation at the 2011 *Australian Association of Mathematics Teachers* conference reported that the first time he engaged with mathematics was when he started on algebra. Indeed there are many topics, including graphing, indices, and introductory algebra that are more accessible than complex fractions and ratios. In other words, schools and teachers should not restrict the curriculum offerings to students based on any preconceptions of their potential. Many schools seem to group students they identify as lower achieving together and teach them these fractions and ratios over and over again, whereas those students would find introductory aspects of other topics more accessible.

One of the unfortunate aspects of mathematics teaching in the middle school is the way that students are streamed and so stereotyped into different levels of achievement. This hardly seems the point of schooling.

In the case of the painting task, the task is already accessible to a range of students since they can choose their own approach to the task and the complexity of the results. Nevertheless there may be some students who will experience difficulty in starting. One approach to supporting such students was described by Sullivan, Mousley, and Zevenbergen (2004) as posing enabling prompts. For example the painting task can be further adapted for students who have difficulty in starting by posing tasks such as the following:

> Ano can paint a house in 5 days. Elizabeth can paint a house in 10 days. How long would it take Ano and Elizabeth to paint a house if they worked together?
>
> Ano can paint a house in 5 days. How much of a house might Ano paint in one day?

The other side of this argument is the importance of extending the more advanced students. Part of the rationale for the practice of streaming off the best students is to preserve their capacity to proceed later to specialist mathematics study. Yet while such practices may be advantageous for some of these students, there are potentially negative effects on others. Hattie (2009), after reviewing large numbers of studies on streaming concluded that "the results show that tracking has minimal effect on learning outcomes and profound negative equity effects" (p. 90). Zevenbergen (2003) also argued that the most commonly observed effect of streaming is reduced opportunities for students in the lower groups.

Rather than streaming students by achievement, the important needs of the higher achieving students can be addressed by focusing on depth of learning rather than breadth and by extending the more advanced students within the content for that level rather than isolating such students into different classes

Sullivan et al. (2004) proposed that teachers pose "extending prompts" to such students. Examples of extending prompts for the painting task might be as follows:

> Ano can paint a house in 3 days. Elizabeth can paint a house in 4 days. Sally can paint a house in 5 days. How long would it take Ano, Elizabeth and Sally to paint a house if they worked together?
>
> Ano can paint 4 houses in 7 days. Elizabeth can paint 3 houses in 5 days. How long would it take Ano and Elizabeth to paint a house if they worked together?

The process for addressing the diversity of achievement in classes and for building equitable outcomes are important foci for teacher learning.

Implications for Teacher Learning

The argument in this chapter is that there are a number of important issues that should be the focus of teacher support and professional learning.

One of the key foci can be on ways of reading the content descriptions, processes for seeking advice or information if anything is unclear, and how the descriptions can be used to support planning and teaching. It seems that we know quite little about the various ways that teachers plan their teaching and assessment and so structured professional learning on the planning options would be helpful. Connected to this is support on ways of using the flexible web format effectively to support and enhance planning.

As argued above, the proficiencies represent an important shift in emphases on mathematical actions. While teachers are familiar with processes for developing fluency and building understanding, they may need support on the development of problem solving and the integration of problem solving processes into the core of the curriculum. This is especially true for reasoning, which will also have some pedagogical implications in that creating opportunities for student reasoning may require a different lesson format from what most teachers are used to.

Teachers will also need support on ways to teach mathematics to heterogeneous groups, including differentiating the experience to optimise the learning of all students within the context of whole class as a community. Of course, this is a major challenge whatever the curriculum, but this new curriculum makes the intention to include all students explicit and so provides an opportunity to engage teachers in professional learning on such issues.

While it has not been mentioned above, there are some substantial changes to the curriculum expectations especially related to statistics and many teachers will need support to understand the rationale for greater emphasis on statistics and statistical literacy, and on the meaning and implications of many of the content descriptions. For example, the content descriptions for data representation and interpretation in year 5 are:

- Pose questions and collect categorical or numerical data by observation or survey
- Construct displays, including column graphs, dot plots and tables, appropriate for data type, with and without the use of digital technologies
- Describe and interpret different data sets in context

The equivalent descriptions for year 7 are:

- Identify and investigate issues involving continuous or large count data collected from primary and secondary sources
- Construct and compare a range of data displays including stem-and-leaf plots and dot plots
- Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data
- Describe and interpret data displays and the relationship between the median and mean

These, and the descriptions for the other levels, may require some interpretation for many teachers. There is a related issue that is associated with the sufficient level of pedagogical knowledge for teachers and particularly that they know how to learn new content, or even how to use old content in new ways.

A Post Script: The Need for Informed and Timely Debate

Even though the focus of this Chapter has been on the content descriptions, it should be recognised that the curriculum generally is a complex set of interwoven documents all of which contribute to the way the first version of the curriculum is presented.

Whether it was part of the thinking of the various governments that proposed and endorsed the development of the national curriculum, the fundamental argument for its creation is to allow for the development of the best quality resources to support the teaching and learning of mathematics. Clearly Australians working together will be able to progressively develop better curriculum and associated teacher support and resources than if they are working in eight separate jurisdictions. The national specification of the cross curriculum capabilities and general capabilities further contribute to the overall quality of the curriculum.

There is little point at this time in debating the placement of topics in the AC:M. For example, there may be some who feel that the fractions content descriptions presented above go too far, and some who argue that students can learn at a faster rate. There will be important debates to be held after this format has been implemented by teachers, but rather than debating minutiae of the curriculum, the focus of debate should be on the best ways to support teachers.

It is also important to recognise that the current version of the curriculum was developed collaboratively after extensive, indeed exhaustive, consultation. The

"Shape Paper" (ACARA, 2010), which established the underlying principles, was developed by a writing team and sought online and face to face feedback nationally. Subsequently, writers who were predominantly classroom teachers were employed, and an advisory committee formed. There were extensive consultations around the successive drafts, piloting in schools across the nation, mapping of the drafts against the various state curricula, and many other steps beside. The advantage of this process is that a curriculum could be developed which is as familiar as possible to as many teachers, and which can ultimately form the basis for subsequent development. The disadvantage is that the writing was informed by many contributions. In other words, there is a tension between seeking consensus and maximising coherence that should be acknowledged. Indeed this publication is further testimony to the extent to which consultation and collaboration have been part of the process.

The content descriptions above clearly address the key ideas of fractions including equivalence, operations, the different forms of fractions (e.g., as a rational number, as a point on a number line, as an operator), and actions using those fractions. There are obviously many ways of describing these ideas, many different sequences are possible, and their alignment with the year levels is also contestable. Nevertheless it is arguable there is no state-based curriculum that is better at representing these key fractional ideas, that these descriptions do address all of the relevant key fraction ideas, and that the key challenge is to find ways to support teachers in using this curriculum as a prompt to better teaching. Continuing to argue about the detail is unhelpful and the challenge is how to use the AC:M and an opportunity for improvement in the teaching of mathematics.

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